

V.V. Narbut, V.N. Salin, E.P. Shpakovskaya

ECONOMIC STATISTICS

TEXTBOOK

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Narbut, Victoria Victorovna.

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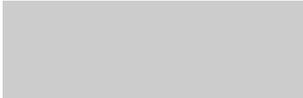
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The monograph was prepared by scientists from the Department of Accounting, Analysis and Auditing of the Financial University under the Government of the Russian Federation, as well as from the Department of Accounting of the National Research University of Nizhny Novgorod. N.I. Lobachevsky, Department of Information and Analytical Support and Accounting, Novosibirsk State University of Economics and Management "NINH", Department of Audit, Accounting and Finance of Novosibirsk State Technical University, Department of Information Systems of the Digital Economy "of the Russian University of Transport (MIIT). Several paragraphs are prepared by masters of the faculty of accounting and auditing of the Financial University under the Government of the Russian Federation. The monograph presents the results of a study on topical issues of ensuring trans-paternity of financial statements of economic entities, including associated with the transition to the FSBU system.

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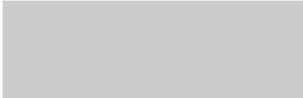


Authors

NARBUT Victoria Vladimirovna — PhD, associate Professor, Department of accounting, analysis and audit of Financial University under the Government of the Russian Federation, graduate of the State University management, Moscow state linguistic University, International Academy of management teachers (Slovenia, track Finance), Berlitz language school (Malta), teacher of international programs at the University of London

SALIN Victor Nikolayevich — PhD, Professor of the Department of accounting, analysis and audit of the Financial University under the Government of the Russian Federation, honored worker of the higher school of the Russian Federation, member of NMS Rosstat, member of the Board of RAS

SHPAKOVSKAYA Elena Petrovna — PhD, associate Professor Department of accounting, analysis and audit of Financial University under the Government of the Russian Federation, honorary worker of higher professional education, laureate of the RF Government prize in education; for active participation in training with the highest qualifications and many years of hard work, the gratitude of the Government of the Russian Federation



Preface

The textbook is intended for students studying economy in English. Statistics is part of the training of an economist of any specialty. Anyone who plans to work in financial institutions, consulting, audit and investment companies, economic services, manufacturing, transport, construction organizations need to be able to work with digital information. The economist should be able to collect, process, summarize information, present it in tabular or graphical form, analyze using statistical methods, interpret the results, draw conclusions, develop a scenario for the development of socio-economic phenomena or processes in the future, to present the results of their work in the form of a report or review so that they were clear to users. Management decisions, external and internal audits, control over the correctness of the calculation of tax liabilities and preparation of tax calculations involve the ability to analyze economic and financial information published in statistical collections, given on information sites, contained in the statistical, accounting and financial statements of economic entities. Digital data skills are essential for market research in goods, financial and non-financial services.

The purpose of the discipline Economic statistics is the formation and development of competencies to perform analytical, applied research and organizational and managerial activity. The textbook is prepared in accordance with the program of teaching statistics in higher educational institutions of financial and economic specialization.

The textbook describes the main statistical methods of collection and presentation and analysis of mass socio-economic data. The textbook can be used both for work in the classroom and self-students studying.

For each section there given learning objectives, definition of basic concepts (in English and Russian), basic formulas, tests, practical tasks and cases.

Practical tasks are based on actual data of Russian Federation Federal State Statistics Service, The Central Bank of Russian Federation, Eurostat, Bloomberg.

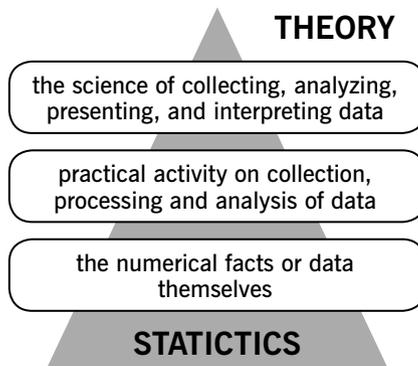
The material of the textbook reflects the features of Russian statistical theory and practice.



Basic definitions and concepts. Types of variables

Learning objectives:

- The definition of Statistics
- The main concepts of Statistics
- Different types of variables



**Subject
of statistics**

quantitative side of events in connection with their qualitative side

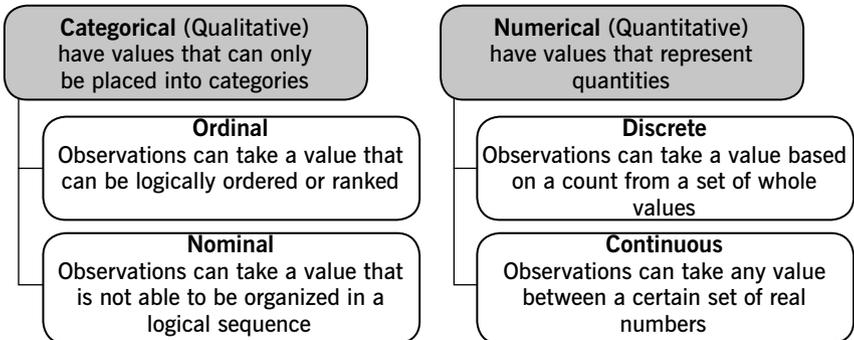
The basic features of Statistics

- deal with population or aggregate of individuals
- studies the quantitative side of events in specific conditions of place and time
- studies facts in statistics and dynamics
- examines the facts in the relationship

Basic definitions

Term / Indicator	Russian equivalent	Definition/ Characteristics
Statistical population	Статистическая совокупность	is a set of homogeneous units of mass phenomenon
Sample	Выборочная совокупность	is a subset of measurements taken from the population in which the researcher is interested
Variable	Статистический показатель	is a characteristic of a unit being observed that may assume more than one of a set of values to which a numerical measure or a category from a classification can be assigned
Value	Величина	is a category that constitute the variable
Statistical data	Статистические данные	refers to data from a survey or administrative source used to produce statistics
Variation	Вариация	is a quantity which may take any of the values of a specified set with a specified relative frequency or probability
Measurement	Измерение	is a process of assigning numbers to characteristics of persons, places, events or things that are observable

Types of variables



TESTS

Test 1-1 (select a correct answer)

Statistics deal with

- 1) Individual facts
- 2) Population or aggregate of individuals
- 3) Rare events
- 4) Significant events

Test 1-2 (select a correct answer)

Statistics is the science concerned

- 1) with the study of the political structure of society
- 2) with developing and studying methods for collecting, analyzing, interpreting and presenting data
- 3) with the construction of accounts
- 4) with the construction graphs.

Test 1-3 (Match the term in the left-hand column with the definition in the right-hand column)

1	Statistical population	a	is a characteristic of a unit being observed that may assume more than one of a set of values to which a numerical measure or a category from a classification can be assigned
2	Value	b	is a process of assigning numbers to characteristics of persons, places, events or things that are observable
3	Variable	c	is a subset of measurements taken from the population in which the researcher is interested
4	Measurement	d	refers to data from a survey or administrative source used to produce statistics
5	Variation	i	is a category that constitute the variable
6	Sample	f	is a quantity which may take any of the values of a specified set with a specified relative frequency or probability
7	Statistical data	g	is a set of homogeneous units of mass phenomenon

Test 1-4 (Match the type of variables in the left-hand column with the definition in the right-hand column)

1	Ordinal	a	Observations can take a value based on a count from a set of whole values
2	Discrete	b	Observations can take a value that can be logically ordered or ranked
3	Continuous	c	Observations can take a value that is not able to be organized in a logical sequence
4	Nominal	d	Observations can take any value between a certain set of real numbers

Test 1-5 (select correct answers)

Select quantitative variables from the list:

- 1) the age of the student
- 2) the gender of the student
- 3) scholarship amount
- 4) field of study (Economic, Management)
- 5) level of education
- 6) the mid-term points
- 7) height

Test 1-6 (select correct answers)

Select qualitative variables from the list:

- 1) the age of the student
- 2) the gender of the student
- 3) scholarship amount
- 4) field of study (Economics, Management)
- 5) level of education
- 6) the mid-term points
- 7) height

PRACTICE

1.1. There is a list of variables:

- 1) Hight
- 2) Number of children in a family
- 3) Clothing size (i.e. small, medium, large, extra-large)
- 4) Gender

- 5) Eye colour
- 6) Form of ownership of the enterprise
- 7) Amount of funds raised by the Bank
- 8) Driving license availability
- 9) Length of service
- 10) Education level.

Divide all variables from the list into four groups:

- a) numerical discrete
- b) numerical continuous
- c) categorical ordinal
- d) categorical nominal.

1.2. For the following table, identify:

- a) Number of observations
- b) Number of variables
- c) Type of each variable (discrete, continuous, nominal, ordinal)

Student	Grade	Age, years	Gender	Height, cm	Class rank
A	3	21	M	183	1
B	4	22	M	181	2
C	2	20	W	160	8
D	3	22	M	179	6
E	4	23	W	165	3
F	1	18	W	173	5
G	2	19	M	185	4

1.3. Given an example of Marketing Research Questionnaire:

Example – Basle Market Survey Questionnaire

1. Are you:

Male

Female

2. What is your age?

18-24

25-34

35-44

45-54

55-64

65 or over

3. What is the highest level of formal education you have completed?
(Please check only one.)

- | | |
|---|--|
| <input type="checkbox"/> Attended High School | <input type="checkbox"/> Graduated High School |
| <input type="checkbox"/> Attended College | <input type="checkbox"/> Graduated College |
| <input type="checkbox"/> Post-Graduate Study Without Degree | <input type="checkbox"/> Post-Graduate Degree |

4. What is your marital status?

- | | |
|--|--|
| <input type="checkbox"/> Married | <input type="checkbox"/> Single. Never Married |
| <input type="checkbox"/> Separated or Divorced | <input type="checkbox"/> Widowed |

5. How many children under the age of 18 live in your household? ___

6. What is your total annual personal income? (Include income from all sources— salary, bonuses, investment income, rents, royalties, etc. Please check only one.)

- | | |
|--|--|
| <input type="checkbox"/> Less than \$30,000 | <input type="checkbox"/> \$30,000 – \$39,999 |
| <input type="checkbox"/> \$40,000 – \$49,999 | <input type="checkbox"/> \$50,000 – \$59,999 |
| <input type="checkbox"/> \$60,000 – \$74,999 | <input type="checkbox"/> \$75,000 – \$99,999 |
| <input type="checkbox"/> \$100,000 – \$149,999 | <input type="checkbox"/> \$150,000 – \$249,999 |
| <input type="checkbox"/> \$250,000 – \$499,999 | <input type="checkbox"/> \$500,000 – \$999,999 |
| <input type="checkbox"/> \$1 million or more | |

7. In which state and ZIP code area is your main residence?

State: _____ ZIP code: _____

8. What is your total annual household income? (Include income for all family members and include all sources—salary, bonuses, investment income, rents, royalties, etc. Please check only one.)

- | | |
|--|--|
| <input type="checkbox"/> Less than \$30,000 | <input type="checkbox"/> \$30,000 – \$39,999 |
| <input type="checkbox"/> \$40,000 – \$49,999 | <input type="checkbox"/> \$50,000 – \$59,999 |
| <input type="checkbox"/> \$60,000 – \$74,999 | <input type="checkbox"/> \$75,000 – \$99,999 |
| <input type="checkbox"/> \$100,000 – \$149,999 | <input type="checkbox"/> \$150,000 – \$249,999 |
| <input type="checkbox"/> \$250,000 – \$499,999 | <input type="checkbox"/> \$500,000 – \$999,999 |
| <input type="checkbox"/> \$1 million or more | |

9a. Do you own a home, condominium as your primary residence?

- | | |
|------------------------------|-----------------------------|
| <input type="checkbox"/> Yes | <input type="checkbox"/> No |
|------------------------------|-----------------------------|

9b. If "Yes,** what is the present market value of your primary residence?

- | | |
|--|--|
| <input type="checkbox"/> Under \$100,000 | <input type="checkbox"/> \$1 million – \$1.9 million |
| <input type="checkbox"/> \$200,000 – \$299,999 | <input type="checkbox"/> \$100,000 – \$199,999 |
| <input type="checkbox"/> \$500,000 – \$749,999 | <input type="checkbox"/> \$300,000 – \$499,999 |

Identify the type of each variable.



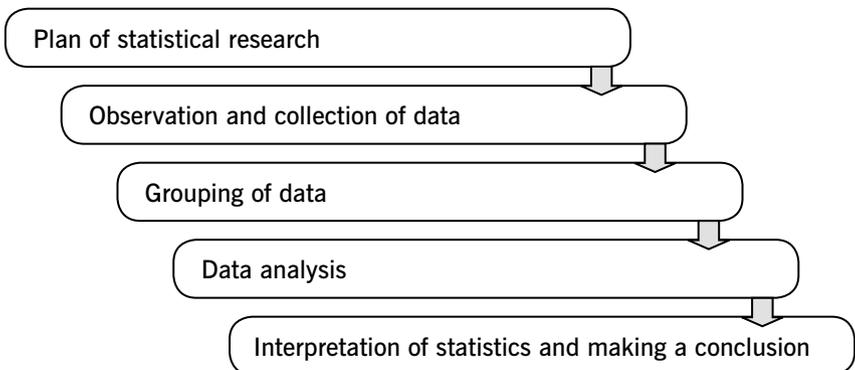
Statistical observation. Grouping of data. Data visualization

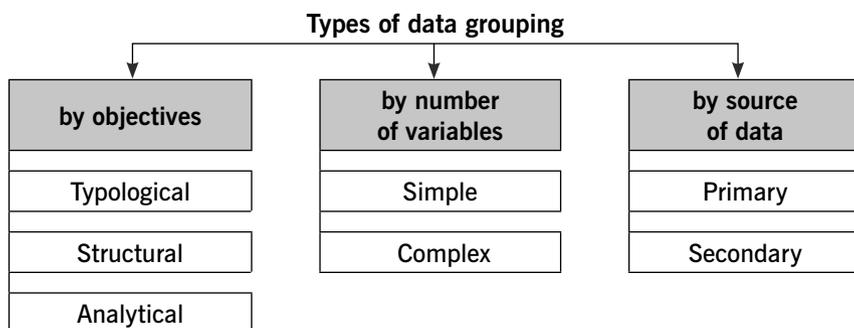
Learning objectives:

- The main steps of statistical observation
- Survey errors
- Types of data grouping
- Grouping categorical and numerical variables
- Visualizing categorical and numerical data

THEORY

Main steps of statistical investigation





Grouping of categorical variables

Grouping tool	Russian equivalent	Characteristic
Summary table	сводная таблица	<ul style="list-style-type: none"> Used for one categorical variable Tallies frequencies or percentages of items in a set of categories
Contingency table	таблица сопряженности	<ul style="list-style-type: none"> Used for several categorical variables Cross tabulates or tallies jointly the responses of the categorical variables

Grouping of numerical variables

Grouping tool	Russian equivalent	Characteristic
Frequency distribution	распределение частот	summary table in which the data are arranged into numerically ordered classes or groups

Main components of frequency distribution

Component	Russian equivalent	Characteristic
Variant	вариант	numeric value of the variable
Frequency	частота	the number of units that have a certain feature
Relative frequency	относительная частота (частость) в долях единицы	frequency expressed in fractions to the total number of observations
Percentage	относительная частота (частость) в процентах	frequency expressed in percent to the total number of observations
Cumulative frequency	накопленная частота	the sum of the frequencies up to a required level

Component	Russian equivalent	Characteristic
Cumulative percentage	накопленная частотъ	the sum of the percentage up to a required level

The number of groups depends on the number of values and its variation.

Types of intervals

Interval	Russian equivalent	Characteristic
Equal	равный	the width of the interval in each group is the same
Unequal	неравный	the width of the interval in each group is the not the same
Opened	открытый	an interval that has only one boundary – upper or lower
Closed	закрытый	an interval that has only two boundary – upper and lower

If the number of groups isn't known, we use **Sturges Rule**:

$$m = 1 + 3,322 \cdot \lg n$$

n – total number of observations.

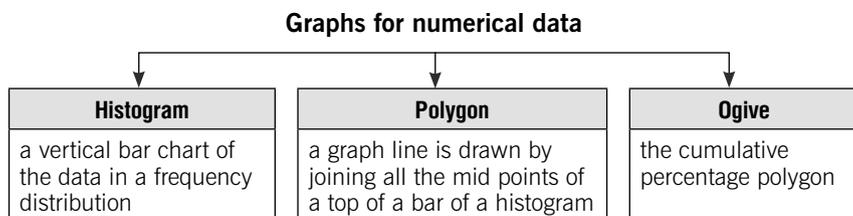
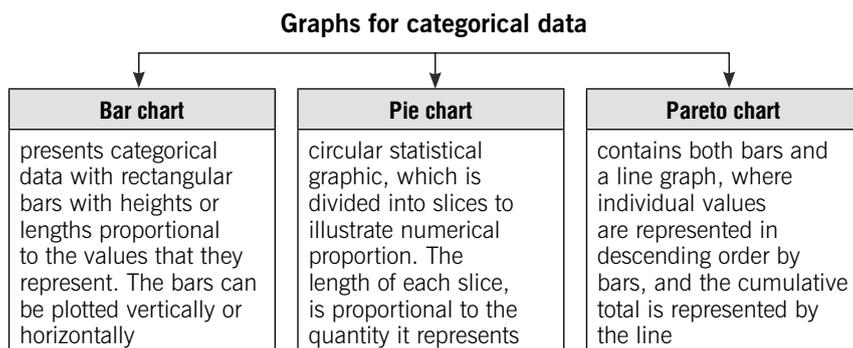
Width (size) of interval

Type of grouping	Width (size) of interval	The symbols in the formula
Grouping with equal intervals	$i = \frac{X_{\max} - X_{\min}}{m}$	X_{\max} – the largest value X_{\min} – the smallest value m – number of groups
Grouping with unequal intervals (arithmetic progression)	$i_k = i_{k-1} + c$	k – number of interval c – constant indicating how much the value of each interval increases
Grouping with unequal intervals (geometric progression)	$i_k = i_{k-1} \cdot c$	k – number of interval c – constant indicating how many times the value of each interval increases

Midpoint (середина интервала)

$$\text{Midpoint} = \frac{\text{Lower boundary} + \text{Upper boundary}}{2}$$

Visualizing categorical data



TESTS

Test 2-1 (set the correct order)

The main steps of statistical investigation:

- 1) Data analysis
- 2) Grouping of data
- 3) Observation and collection of data
- 4) Interpretation of statistics and making a conclusion
- 5) Plan of statistical research

Test 2-2 (select correct answers)

The tasks solved by grouping of data:

- 1) identification of relationship between variables
- 2) define grouping variable
- 3) allocation of social and economic types
- 4) studying of structure
- 5) determine the width of interval

Test 2-3 (select a correct answer)

An interval that has one boundary is called ...

- 1) equal
- 2) unequal
- 3) opened
- 4) closed

Test 2-4 (select a correct answer)

An interval that has two boundaries is called ...

- 1) equal
- 2) unequal
- 3) opened
- 4) closed

Test 2-5 (select a correct answer)

Intervals «100-200» «200-300» «300-400» «400-500»

- 1) equal and opened
- 2) unequal and closed
- 3) equal and closed
- 4) unequal and opened

Test 2-6 (select a correct answer)

Intervals «100-200» «200-400» «400-800» «800-1600»

- 1) equal and opened
- 2) unequal and closed
- 3) equal and closed
- 4) unequal and opened

Test 2-7 (Match the type of data grouping in the left-hand column with the task solved by grouping of data in the right-hand column)

1	Analytical	a	studying of structure
2	Typological	b	identification of relationship between variables
3	Structural	c	allocation of social and economic types

Test 2-8 (select a correct answer)

It is known that the frequency of the first interval is 10, the second – 15, the third – 20. Determine the cumulative frequency of the second interval

- 1) 15
- 2) 20
- 3) 25
- 4) 45

Test 2-9 (select a correct answer)

It is known that the frequency of the first interval is 10, the second – 30, the third – 20. Determine the percentage of the second interval

- 1) 16,6%
- 2) 33,3%
- 3) 50,0%
- 4) 66,7%

Test 2-10 (select a correct answer)

The interval distribution consists of three equal intervals. It is known that the cumulative frequency of the first interval is 10, the second – 30, the third – 60. Determine percentage of the third interval

- 1) 100,0%
- 2) 50,0%
- 3) 83,3%
- 4) 60,0%

Test 2-11 (count and write the correct answer)

There are 50 observations. Determine the number of groups in the frequency distribution using the Sturges formula

Test 2-12 (count and write the correct answer)

It is known that the minimum value is equal 50, and the maximum – 200. It is planned to construct an interval distribution consists of five groups. Determine the width of the interval

Test 2-13 (select a correct answer)

The interval value is calculated

- 1) as a difference between upper and lower boundaries
- 2) as a sum of upper and lower boundaries
- 3) according to the formula of arithmetic mean
- 4) according to the Sturges formula

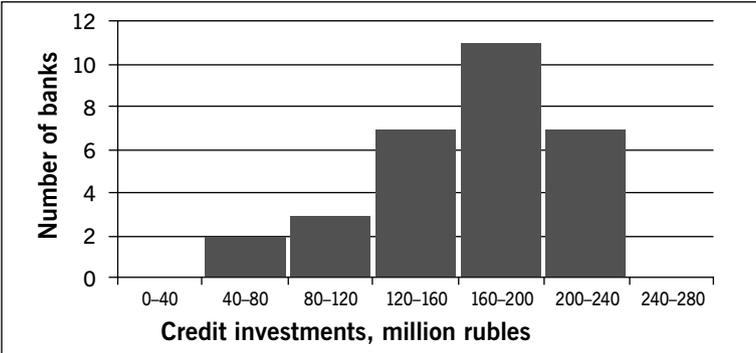
Test 2-14 (select a correct answer)

Polygon is ...

- 1) a vertical bar chart of the data in a frequency distribution
- 2) a graph line is drawn by joining all the mid points of a top of a bar of a histogram
- 3) circular statistical graphic, which is divided into slices to illustrate numerical proportion
- 4) chart of categoric data with rectangular bars with heights or lengths proportional to the values that they represent

Test 2-15 (select a correct answer)

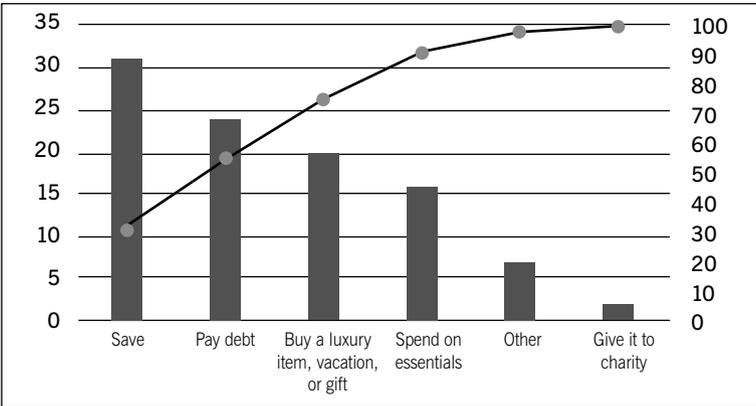
Specify the type of the graph



- 1. Histogram
- 2. Polygon
- 3. Bar chart
- 4. Pie chart

Test 2-16 (select a correct answer)

Specify the type of the graph



- 1) Histogram
- 2) Ogive
- 3) Bar chart
- 4) Pareto chart

Test 2-17 (select correct answers)

Select variables by which summary table may can be constructed:
the age of the student

- 1) the gender of the student
- 2) scholarship amount

- 3) field of study (Economics, Management)
- 4) level of education
- 5) the mid-term points
- 6) height

Test 2-18 (select correct answers)

Select variables by which frequency distribution may can be constructed:

- 1) the age of the student
- 2) the gender of the student
- 3) scholarship amount
- 4) field of study (Economics, Management)
- 5) level of education
- 6) the mid-term points
- 7) height

PRACTICE

2.1. The total population in the Russian Federation is the following (for the beginning of year, million persons): 2013 – 143,3; 2014 – 143,7; 2015 – 146,3; 2016 – 146,5; 2017 – 146,8.

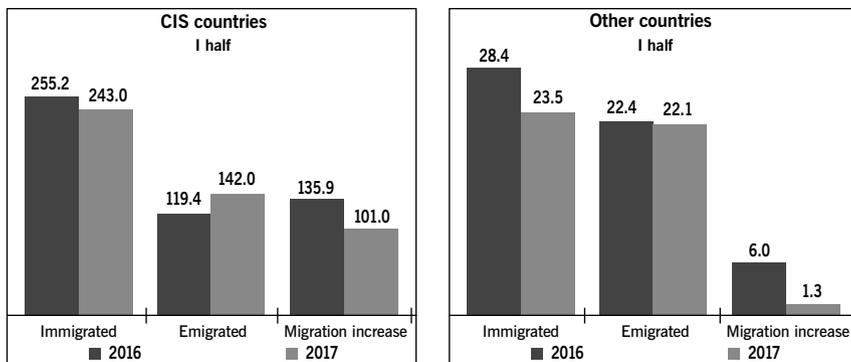
It is known that the percent of urban population for every year is 75% of all population.

Perform the following tasks:

Present all the data in the statistical table which must include the number of all population, urban and rural population.

2.2. The following two graphs are available:

Russia – Foreign Countries Migration of Population (thousand persons)



Perform the following tasks:

Present data from graphical Rosstat presentation «Russia-Foreign countries migration of population» in two statistical tables:

1. Number of migrants, arrived in Russia (total, from CIS countries, from other countries) in 2016 and 2017.

2. Number of migrants, left Russia (total, to CIS countries, to other countries) in 2016 and 2017.

2.3. There are data of the unemployment persons and number of the months they were without work:

№ of persons	Male / Female	Number of months	№ of persons	Male / Female	Number of months
1	M	2	13	M	2
2	F	4	14	M	3
3	M	3	15	F	4
4	M	1	16	M	1
5	F	3	17	F	3
6	M	2	18	M	2
7	F	3	19	M	3
8	F	2	20	M	2
9	M	1	21	F	2
10	M	3	22	F	4
11	F	2	23	M	1
12	M	2	24	F	3

Perform the following tasks:

Group the unemployed persons:

- 1) by gender;
- 2) by the number of months without work;
- 3) by gender and the number of months without work.

2.4. Given the data about assessment of activity of the company by the 63 clients in points from 1 to 6:

4	6	5	2	5	5	5	5	5
4	5	5	3	2	3	2	4	4
3	2	6	2	4	4	5	4	6

4	3	3	6	2	6	4	3	4
5	5	5	1	4	5	5	1	6
1	6	4	6	5	4	5	6	5
4	3	5	5	5	4	3	1	6

Perform the following tasks:

1. Group the data.
2. Calculate frequencies, cumulative frequencies and percentage for each group.
3. Construct a polygon.

2.5. Given the number of the staff of organizations:

190	148	182	134	70	104	135	121
125	42	136	178	147	159	103	170
145	104	119	146	138	90	82	160
74	85	44	78	141	110	79	85
198	129	165	175	112	137	139	170
173	57	86	157	138	118	120	180
167	200	45	210	228	168	99	112

Perform the following tasks:

1. Construct an interval distribution forming 6 groups.
2. Calculate frequencies, cumulative frequencies and percentage for each group.
3. Construct histogram, polygon and ogive.

2.6. Given the data on length of service and the salary of 20 employees of firm:

N _e	Length of service, years	Monthly salary, rubles	N _e	Length of service, years	Monthly salary, rubles
1	10	43200	11	8	42000
2	2	21300	12	6	35800
3	3	25600	13	11	42000
4	12	42700	14	5	37200
5	5	32700	15	15	46000

№	Length of service, years	Monthly salary, rubles	№	Length of service, years	Monthly salary, rubles
6	13	43000	16	4	30000
7	5	29900	17	7	40000
8	10	41900	18	6	39000
9	8	41400	19	9	42200
10	7	39100	20	2	22000

Perform the following tasks:

1. Group employees on length of service and make three groups. For each group calculate number of persons and average monthly salary.

2. Make a conclusion about relationship between the length of service and the salary.

2.7. Group the countries on expenses on health care per capita. The number of groups define by Sturges formula.

№	Country	Expenses on health care per capita, \$	№	Country	Expenses on health care per capita, \$
1	Australia	3150	16	Italy	2430
2	Austria	3450	17	Canada	3200
3	Argentina	1280	18	Luxembourg	4230
4	Bahamas	1360	19	Malta	1750
5	Belgium	3100	20	Netherlands	3120
6	Brazil	1500	21	New Zealand	2100
7	United Kingdom	2580	22	Norway	4100
8	Hungary	1320	23	Portugal	1900
9	Germany	3200	24	Slovenia	1800
10	Greece	2200	25	Finland	2200
11	Denmark	2800	26	France	3070
12	Israel	1990	27	Czech Republic	1400
13	Ireland	2640	28	Switzerland	4050
14	Iceland	3320	29	Sweden	2850
15	Spain	2120	30	Japan	2300

2.8. Given the data on the construction companies of the region on the ownership type and number of employees:

№	Ownership type	Number of employees	№	Ownership type	Number of employees
1	state and municipal	90	15	private	68
2	private	170	16	state and municipal	80
3	state and municipal	80	17	private	50
4	private	110	18	other	80
5	private	160	19	private	104
6	other	100	20	private	40
7	private	63	21	private	55
8	private	75	22	state and municipal	94
9	private	72	23	private	154
10	private	80	24	private	63
11	private	65	25	private	73
12	other	90	26	private	110
13	private	72	27	private	88
14	state and municipal	60	28	state and municipal	70

Perform the following tasks:

1. Group the companies by ownership type. Calculate the number of companies and the number of employees per 1 company for each ownership type. Draw a pie chart.

2. Make the interval distribution with the interval on number of employees: «under 70», «70-110», «110 and more».

3. Group the companies by two variables – ownership type and the number of employees. The results of the group present in one table. Specify the grouping type by the two characteristics.

2.9. Given the data on the salary of 22 employees of firm:

№	Monthly salary, thousand rubles	№	Monthly salary, thousand rubles
1	57,3	12	42,0
2	21,9	13	30,8
3	23,6	14	44,0

Окончание

№	Monthly salary, thousand rubles	№	Monthly salary, thousand rubles
4	46,7	15	78,5
5	26,7	16	31,4
6	50,0	17	58,1
7	24,1	18	26,1
8	41,9	19	39,0
9	41,4	20	33,8
10	77,8	21	45,2
11	34,5	22	22,4

Group employees on the salary and make groups with unequal intervals:

1. arithmetic progression, with a constant equal to 4, the first group «under 22000», the second « 22 – 26»;

2. geometric progression, with a constant equal to 2, the first group «under 22», the second «22 – 24».

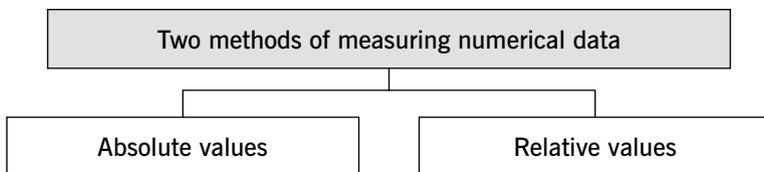


Absolute and Relative Values

Learning objectives:

- Methods of measuring numerical data
- Absolute values
- Types of relative values

THEORY



Absolute values are the real/precise numbers.

Measurement units of absolute values:

- natural (kilograms, tons, meters, liters, pieces, etc.)
- monetary (rubles, euros, dollars, etc.)
- labour (man-days, man-hours)

Relative values are dependent on absolute values.

$$\text{Relative value} = \frac{\text{Absolute value 1}}{\text{Absolute value 2}}$$

Expressed in:

- coefficient
- percentage – 1 part in 100 (%)
- per mile – 1 part in 1000 (‰)
- per decimals – 1 part in 10000 (‱)
- mixed units of measure

Types of relative values

Relative value	Russian equivalent	Method of calculation and characteristics
<p>Ratio of plan target</p> <p>Characterizes the change of the planned level in comparison with the previous level</p>	Относительная величина планового задания	$RPT = \frac{y_{pl}}{y_0} \cdot 100 \%$ <p>y_{pl} – planned absolute value y_0 – absolute value in previous period Expressed in:</p> <ul style="list-style-type: none"> • coefficient • percentage (%)
<p>Ratio of plan implementation</p> <p>Characterizes the change of the actual level of the variable in comparison with the plan level</p>	Относительная величина выполнения плана	$RPI = \frac{y_1}{y_{pl}} \cdot 100 \%$ <p>y_{pl} – planned absolute value y_1 – absolute value in current period Expressed in:</p> <ul style="list-style-type: none"> • coefficient • percentage (%)
<p>Dynamic Ratio</p> <p>Characterizes the change of current level in comparison with the previous level</p>	Относительная величина динамики	$DR = \frac{y_1}{y_0} \cdot 100 \%$ $DR = RPT \cdot RPI$ <p>y_0 – planned absolute value y_1 – absolute value in current period Expressed in:</p> <ul style="list-style-type: none"> • coefficient • percentage (%)
<p>Structure Ratio</p> <p>Characterizes the share of the whole</p>	Относительная величина структуры	$SR = \frac{\text{Part of total}}{\text{Total}} \cdot 100 \%$ <p>Expressed in:</p> <ul style="list-style-type: none"> • coefficient • percentage (%)

Окончание

Relative value	Russian equivalent	Method of calculation and characteristics
<p>Coordination Ratio</p> <p>Characterizes the ratio of two parts of the whole</p>	Относительная величина координации	$Ratio_{coord} = \frac{\text{Part 1 of total}}{\text{Part 2 of total}}$ <p>Expressed in:</p> <ul style="list-style-type: none"> • 1 part in 10 • 1 part in 100 • 1 part in 1000
<p>Ratio of comparison</p> <p>Characterizes the ratio of absolute values of different objects or territories for the same period of time</p>	Относительная величина сравнения	$Ratio_{comp} = \frac{\text{Absolute value of object A}}{\text{Absolute value of object B}}$ <p>Expressed in: coefficient percentage (%)</p>
<p>Ratio of intensity</p> <p>Characterizes the spread of the phenomenon in its environment</p>	Относительная величина интенсивности	$Ratio_{intensity} = \frac{\text{Absolute value of object}}{\text{Absolute value of the spread of the object}}$ <p>Expressed in mixed units of measure</p>

TESTS

Test 3-1 (select incorrect answers)

Absolute values cannot be expressed in:

- 1) natural units
- 2) percentage
- 3) monetary units
- 4) coefficient
- 5) labour units

Test 3-2 (select incorrect answers)

Relative statistics cannot be expressed in:

- 1) natural units
- 2) percentage
- 3) monetary units
- 4) coefficient
- 5) labour units

Test 3-3 (select correct answers)

Select the ratio of the comparison:

- 1) the population of region A was 1 million people, and region B – 2 million people
- 2) the population of region A is 2 times more than region B
- 3) the region's population A is 250 thousand more than in the region B
- 4) the share of the female population in region A is 54 %, and in region B – 51 %
- 5) the population in region A is 20% more than in the in the region B

Test 3-4 (select a correct answer)

Specify the structure ratio:

- 1) the population of region A was 2 million people, and region B – 5 million people
- 2) the population of region A is 3 times more than region B
- 3) the population of region A is 250 thousand more than in the region B
- 4) the share of the female population in region A is 54 %, and in region B – 51 %
- 5) the population in region A is 10% more than in the in the region B

Test 3-5 (select a correct answer)

The sum of the structure ratios calculated for all groups of units of the population should be ...

- 1) less than 100%
- 2) more than 100%
- 3) equal to 100%
- 4) less than 100% or more than 100%

Test 3-6 (select a correct answer)

It is known that the proportion of men in the population is 48%. It is the relative value

- 1) ratio of comparison
- 2) structure ratio
- 3) coordination ratio
- 4) dynamic ratio
- 5) ratio of intensity

Test 3-7 (Match the type of relative value in the left-hand column with its characteristic in the right-hand column)

1	Ratio of intensity	a	Characterizes the change of the planned level in comparison with the previous level
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2	Ratio of plan target	b	Characterizes the ratio of two parts of the whole
3	Structure Ratio	c	Characterizes the change of the actual level of the variable in comparison with the plan level
4	Ratio of comparison	d	Characterizes the share of the whole
5	Ratio of plan implementation	e	Characterizes the change of current level in comparison with the previous level
6	Coordination Ratio	f	Characterizes the spread of the phenomenon in its environment
7	Dynamic Ratio	g	Characterizes the ratio of absolute values of different objects or territories for the same period of time

Test 3-8 (select a correct answer)

Select the correct statement

- 1) The ratio of plan target if the product of ratio of plan implementation and dynamic ratio
- 2) The dynamic ratio is the product of ration of plan target and ratio of plan implementation
- 3) The ratio of plan implementation is the product of ration of plan target and dynamic ratio

Test 3-9 (select a correct answer)

Planned absolute value = 200 thousand rubles

Absolute value in previous period = 150 thousand rubles

Absolute value in current period = 250 thousand rubles

Ratio of plan implementation is equal to:

1. 125%
2. 133%
3. 167%
4. 75%

Test 3-10 (select a correct answer)

Planned absolute value = 200 thousand rubles

Absolute value in previous period = 150 thousand rubles

Absolute value in current period = 250 thousand rubles

Ratio of plan target is equal to:

- 1) 125%
- 2) 133%
- 3) 167%
- 4) 75%

PRACTICE

3.1. It is known that 14,6 million tablets have been sold in the I quarter 2018. According to the analytical company ICD it was planned that the sales volume of tablets will be 19,2 million in the II quarter 2018. However actually it was succeeded to sell only 18,1 million.

Define:

1. Ratio of plan target.
2. Dynamic ratio.
3. Ratio of plan implementation.

3.2. According to Institute of the market researches Gartner in 2014 it was planned to increase sale of tablets by 25% in comparison with 2013 and to sale 256 million. However actually in 2014 have produced only 216 million devices.

Define:

1. Ratio of plan implementation
2. Dynamics ratio
3. Number of the sold tablets in 2013.

3.3. According to the *IDC Quarterly Mobile Phone Tracker top five smartphones vendors are the following:*

Top Five Smartphone Vendors

Vendor	Shipment volume, million		Market share of the smartphone vendors, %		Growth 2017/2016, %
	I Quarter 2016	I Quarter 2017	I Quarter 2016	I Quarter 2017	
Samsung	79,2	79,2			
Apple	51,2	51,6			
Huawei	28,1	34,2			
OPPO	19,7	25,6			
Vivo	14,6	18,1			
Others	140,0	138,7			
Total:	332,8	347,4			

Source: IDC Quarterly Mobile Phone Tracker, April 27, 2017

Perform the following tasks:

1. Find market share of the smartphone vendors in the I quarter 2016 and the I quarter 2017 using structure ratio (accuracy 0,1).

2. Find growth of every vendor's shipment volume and the total shipment volume in 2017 comparing to 2016 using dynamics ratio (accuracy 0,1).

3. Present the market share of the smartphone vendors in the I quarter 2016 and the I quarter 2017 using pie chart.

3.4. Territory and urban settlements of constituent entities in Central Federal District of Russian Federation on January 1, 2017 are the following:

**Territory and urban settlements of constituent entities
in Central Federal District**

Region	Territory, thousand sq. km	Population, thousand persons	Number of inhabitants per 1 sq. km
Belgorod Region	27,1	1552,9	
Bryansk Region	34,9	1220,5	
Vladimir Region	29,1	1389,6	
Voronezh Region	52,2	2335,4	
Ivanovo Region	21,4	1023,2	
Kaluga Region	29,8	1014,6	
Kostroma Region	60,2	648,2	
Kursk Region	30,0	1122,9	
Lipetsk Region	24,0	1156,2	
Moscow Region	44,3	7423,5	
Orel Region	24,7	754,8	
Ryazan Region	39,6	1126,7	
Smolensk Region	49,8	953,2	
Tambov Region	34,5	1040,3	
Tver Region	84,2	1296,8	
Tula Region	25,7	1499,4	
Yaroslavl Region	36,2	1270,7	
Moscow city	2,6	12380,7	

Source: Russian statistical yearbook. 2017

Perform the following tasks:

1. Compare regions of the Central Federal District of Russian Federation on the number of inhabitants per 1 sq. km.

2. Range the regions by the number of inhabitants per 1 sq. km.

3.5. There are data on population of the countries of the world on January 1, 2016:

Number of Males and Females

Country	Total, thousand persons		Share of total population, %		Number of females per 100 males
	males	females	males	females	
Russia	68044	78760			
India	649374	604644			
USA	158229	163190			
France	31283	33230			

Source: Russian statistical yearbook. 2017

Find for each country:

1. Share of total population by gender (%).
2. Number of females per 1000 males.

3.6. There are following data on capitalization of the Russian companies (February, 2015):

Company	Capital, billion US dollars
Gazprom	53,95
Rosneft	42,57
Sberbank	21,28

Compare the capital of the Russian companies with the capital of Apple, the most expensive company in the world, which capital 727,4 bln. US dollars.

3.7. There are data on export and import of goods (billion US dollars):
Export and Import of Goods, 2016

EC Countries	Exports	Imports	Proportion of foreign trade turnover, %	
			Exports	Imports
Austria	145	149		
Belgium	399	372		
Bulgaria	26	28		
Hungary	103	92		
Germany	1335	1056		

Окончание

EC Countries	Exports	Imports	Proportion of foreign trade turnover, %	
			Exports	Imports
Denmark	94	85		
Italy	455	401		
Lithuania	25	27		
Netherlands	511	421		
Poland	202	197		
Romania	63	74		
United Kingdom	407	588		
Finland	57	60		
France	488	560		
Sweden	139	140		

Source: Russian statistical yearbook. 2017

The total value of export and import of the country is equal its foreign trade turnover.

Perform the following tasks:

1. Define the proportion of foreign trade turnover for each country.
2. Range EU countries by export share of foreign trade turnover.

3.8. There are data on electricity generation by the countries of the world for 2010:

Production of electricity in 2010

Country of the world	Electricity, bln. kWh	Population, mln. persons	Electricity per capita
Russia	1038	142,9	
Australia	261	22,3	
Germany	592	81,9	
China	3696	1338,0	
France	542	63,0	

Source: Russian statistical yearbook. 2017

Perform the following tasks:

1. Range the countries by total amount of electricity.
2. Define electricity per capita for each country.
3. Range the countries by electricity per capita.

3.9. Given the indicators characterizing marriages and divorces of population in 2015:

Marriages and divorces

Indicator	Moscow city	Tyumen Region	Chechen Republic
Marriages	99 873	32 174	7 946
Divorces	44 388	21346	1 115
Population, thousand persons	12 330	3 615	1 394
Territory, thousand square km	2,6	1464,2	15,6

Define and compare performance by region:

1. Marriages per 1000 persons
2. Divorces per 1000 persons
3. Divorces per 100 marriages.
4. Population density.

3.10. Employees of federal executive power bodies of some countries of the world are the following:

Employees of federal executive power bodies of some countries of the world, 2016

Country	Employees of federal executive power bodies, thou. persons	Population, mln. persons	Number of employed, mln. persons
Russia	1203	146,8	72,4
United Kingdom	370	65,4	31,6
USA	2079	323,0	151,4

Define:

1. Number of employees of federal executive power bodies per 1000 population.
2. Number of employees of federal executive power bodies per 1000 employed.

3.11. Given the data on dynamics of retail trade turnover of food products and nonfood goods (billion rubles):

Trade turnover

Period	Food products			Nonfood goods		
	2017	2018	2018 / 2017, %	2017	2018	2018 / 2017, %
October	973,4		110,6		1213,6	

Окончание

Period	Food products			Nonfood goods		
	2017	2018	2018 / 2017, %	2017	2018	2018 / 2017, %
November		1091,8	110,4	1110,5		111,2
December	1189,7	1360,0		1327,5	1581,8	
IV quarter				3547,7		

Put down the missing data in the table.

3.12. There are following data on structure of income of the budget of Moscow:

billion rubles

Type of income	2016		2017	
	Approved by law	Actual execution	Approved by law	Actual execution
Tax revenues	1363,2	1 403,2	1 755,2	1 795,8
Non-tax revenues	165,7	175,1	229,9	254,1
Revenues total	1 598,9	1 649,5	1 997,0	2 097,0

The approved total revenues of the budget of St. Petersburg for 2017 is 502,9 billion rubles. The average population of Moscow in 2016 was 12 381 thousand people. The population of Moscow consumed in 2016 meat and meat products 1 189 thousand tons, milk and dairy products 3 120 thousand tons. Area of Moscow – 2 561 sq. km.

Perform the following tasks:

- 1) Define all kinds of relative values.
- 2) Comment on the results.

UNIT 4



Measures of Central Tendency

Learning objectives:

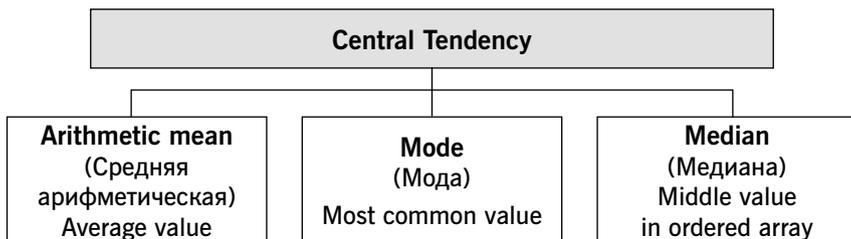
- Mean
- Mode
- Median
- Quartile measures
- Deciles

THEORY

Basic definitions

Indicator	Russian equivalent	Definition/ Characteristics
Harmonic mean	Средняя гармоническая	Average value
Geometric mean	Средняя геометрическая	Average value
Arithmetic mean	Средняя арифметическая	Average value
Quadratic mean	Средняя квадратическая	Average value
Mode	Мода	Most common value
Median	Медиана	Middle value in ordered array
Quartile	Квартиль	Indicator that split the ranked data into 4 parts with an equal number of values in each part
Decile	Дециль	Indicator that divide the data into ten equal parts

Measures of central tendency



Arithmetic mean

1. Most common measure of central tendency
2. Affected by extreme values

	For ungrouped data	For grouped data
Formula	$\bar{x} = \frac{\sum X}{n}$ <p>x = value n = number of values</p>	$\bar{x} = \frac{\sum X \cdot f}{\sum f}$ <p>x = value f = frequency</p>

Types of Mean

Mean	Formula	
	Simple (for ungrouped data)	Weighted (for grouped data)
Harmonic	$\bar{x} = \frac{n}{\sum \frac{1}{x}}$	$\bar{x} = \frac{\sum F}{\sum \frac{F}{x}}$
Geometric	$\bar{x} = \sqrt[n]{x_1 \cdot x_2 \cdot \dots \cdot x_n}$	$\bar{x} = \sqrt[\sum f]{x_1^{f_1} \cdot x_2^{f_2} \cdot \dots}$
Arithmetic	$\bar{x} = \frac{\sum X}{n}$	$\bar{x} = \frac{\sum X \cdot f}{\sum f}$
Quadratic	$\bar{x} = \sqrt{\frac{\sum X^2}{n}}$	$\bar{x} = \sqrt{\frac{\sum X^2 \cdot f}{\sum f}}$

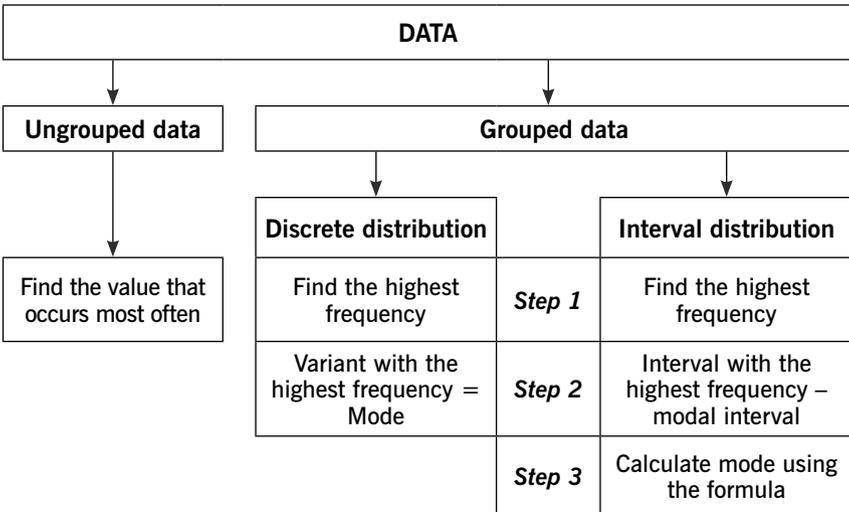
The General formula of the exponential average

Simple (for ungrouped data)	Weighted (for grouped data)	Symbols in the formula
$\bar{x} = \sqrt[k]{\frac{\sum x_i^k}{n}}$	$\bar{x} = \sqrt[k]{\frac{\sum x_i^k f_i}{\sum f_i}}$	x_i – value f_i – frequency n – number of values k – power
$k = -1$ harmonic mean		
$k = 0$ geometric mean		
$k = 1$ arithmetic mean		
$k = 2$ quadratic mean		

Mode

1. Value that occurs most often
2. Not affected by extreme values
3. There may be no mode or two (or three) modes

Steps of calculating mode

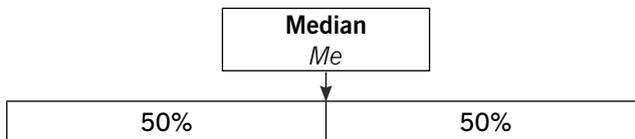


Formula for calculating mode in interval distribution

Formula	Symbols in the formula
$MO = x_{Mo} + i \frac{f_{Mo} - f_{Mo-1}}{(f_{Mo} - f_{Mo-1}) + (f_{Mo} - f_{Mo+1})}$	x_{Mo} – lower boundary of modal interval i – width (size) of modal interval f_{Mo} – frequency of modal interval f_{Mo-1} – frequency of interval before modal interval f_{Mo+1} – frequency of the interval after modal interval

Median

1. In ordered array, the median is the middle value (50% above and 50% below)
2. Not affected by extreme values



Steps of calculating median

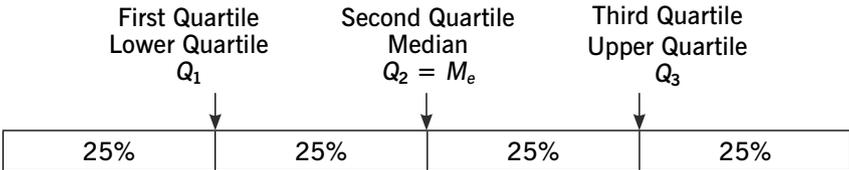
DATA			
↓	↓		
Ungrouped data	Grouped data		
↓	↓	↓	
Step 1 Order the data	Discrete distribution	Step 1	Interval distribution
Step 2 Define the median position $N_{Me} = \frac{n+1}{2}$	Define the median position $N_{Me} = \frac{\sum f}{2}$	Define the median position $N_{Me} = \frac{\sum f}{2}$	Define the median position $N_{Me} = \frac{\sum f}{2}$
Step 3 If n is odd, then median is middle number If n is even, then median is the average of middle numbers	Define cumulative frequencies	Step 2	Define cumulative frequencies
	Find the cumulative frequency greater than the number of the median	Step 3	Find the cumulative frequency greater than the number of the median
	Value with this cumulative frequency = median	Step 4	Interval with this cumulative frequency = median interval
		Step 5	Calculate median using the formula

Formula for calculating median in interval distribution

Formula	The symbols in the formula
$M_e = x_{Me} + i \frac{\frac{\sum f}{2} - Cf_{Me-1}}{f_{Me}}$	x_{Me} – lower boundary of median interval i – width of median interval f_{Me} – frequency of median interval Cf_{Me-1} – cumulative frequency of interval before median interval

Quartile measures

Quartiles split the ranked data into 4 parts with an equal number of values in each part



Interpretation of quartile measures

Quartile	Interpretation
First quartile Q_1	The value for which 25% of the observations are smaller and 75% are larger
Second quartile Q_2	The value for which 50% of the observations are smaller and 50% are larger
Third quartile Q_3	The value for which 25% of the observations are larger

Determining quartiles for ungrouped data

- Order the values from the smallest to the largest
- Define the position of quartiles $N_{Q_1} = \frac{n+1}{4}$; $N_{Q_2} = \frac{2(n+1)}{4}$; $N_{Q_3} = \frac{3(n+1)}{4}$
- Define quartiles according to their position

Determining quartiles for grouped data

- Define the position of quartiles $N_{Q_1} = \frac{\sum f}{4}$; $N_{Q_2} = \frac{2 \cdot \sum f}{4}$; $N_{Q_3} = \frac{3 \cdot \sum f}{4}$
- Define cumulative frequencies
- Find the cumulative frequency greater than the number of the quartiles
- Define quartile (for discrete distribution) or quartile interval (for interval distribution)
- Calculate quartiles using the formula

Quartiles for interval distribution

$Q = x_Q + i \frac{N_Q - Cf_{Q-1}}{f_Q}$	x_Q – lower boundary of quartile interval i – width (size) of quartile interval f_Q – frequency of quartile interval Cf_{Q-1} – cumulative frequency of the interval before quartile interval
$N_{Q_1} = \frac{\sum f}{4}; N_{Q_2} = \frac{2 \cdot \sum f}{4}; N_{Q_3} = \frac{3 \cdot \sum f}{4}$	

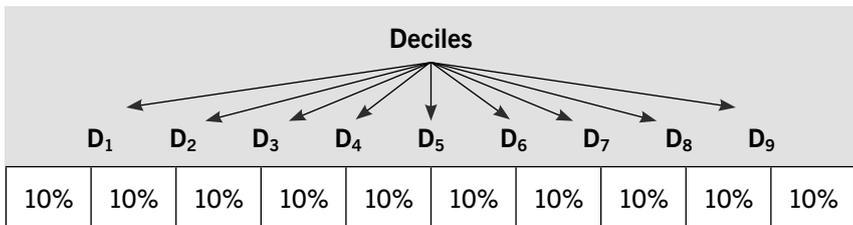
The interquartile range (IQR)

The interquartile range measures the spread in the middle 50% of the data. The IQR is a measure of variability that is not influenced by extreme values.

Formula $IQR = Q_3 - Q_1$

Decile measures

Deciles divide the data into ten equal parts. The deciles can determine the values for 10%, 20%, 30%.....90% of the data



Determining deciles for ungrouped data

1. Order the values from the smallest to the largest
2. Define the position of deciles $N_{D_1} = \frac{n+1}{10}, \dots, N_{D_9} = \frac{9(n+1)}{10}$
3. Define deciles according to their position

Determining deciles for grouped data

1. Define the position of quartiles $N_{D_1} = \frac{\sum f}{10}, \dots, N_{D_9} = \frac{9 \cdot \sum f}{10}$
2. Define cumulative frequencies
3. Find the cumulative frequency greater than the number of the deciles
4. Define deciles (for discrete distribution) or decile interval (for interval distribution)
5. Calculate deciles using the formula

Deciles for interval distribution

$D = x_D + i \frac{N_D - Cf_{D-1}}{f_D}$	x_D – lower boundary of decile interval i – width (size) of quartile interval f_D – frequency of decile interval
$N_D = \frac{k \cdot \sum f}{4};$	Cf_{D-1} – cumulative frequency of the interval before decile interval
$k = 1, 2, 3, \dots, 9$	N_D – decile number

Decile ratio

It concerns the distance between two ends of the distribution. It is commonly used for measuring income inequalities (distance between the richest and the poorest)

Formula $Ratio_D = \frac{D_9}{D_1}$

TESTS

Test 4-1 (select a correct answer)

Median is the

1. Most common value
2. Average value
3. Middle value
4. Value, ranked data into 4 parts

Test 4-2 (select a correct answer)

Quartile is the

1. Most common value
2. Average value
3. Middle value
4. Value, ranked data into 4 parts

Test 4-3 (select a correct answer)

There is a set of data: 10 12 14 9 7 11 12 8 12.
The mode is equal to

- | | |
|-------|------------|
| 1. 14 | 3. 7 |
| 2. 12 | 4. No mode |

Test 4-4 (select a correct answer)

There is a set of data: 10 12 14 9 7 11 12 8 12.

The number of the median is equal to

1. 5,0
2. 4,5
3. 4,0
4. 9,0

Test 4-5 (select a correct answer)

There is a set of data: 10 12 14 9 7 11 12 8 12.
The median is equal to

1. 7
2. 10
3. 11
4. 12

Test 4-6 (select a correct answer)

There is a set of data: 10 12 14 9 7 11 12 8 12.
The third quartile is equal to

1. 7
2. 10
3. 11
4. 12

Test 4-7 (select a correct answer)

Given the following distribution:

Size of the family	2	3	4	5	6
Number of families, thousand	14	16	12	8	5

The mode is equal to

1. 14
2. 16
3. 3
4. 4

Test 4-8 (select a correct answer)

Bank deposit, thousand rubles	Up to 40	40 - 60	60 - 80	80 - 100	100 and more
Number of the depositors	120	280	100	80	40

The cumulative frequency of the interval "60-80" is

1. 280
2. 400
3. 100
4. 500

Test 4-9 (set the correct order)

The steps of calculating median for interval distribution are:

1. Find the cumulative frequency greater than the number of the median
2. Find the cumulative frequency greater than the number of the median
3. Define the median position

4. Define cumulative frequencies

5. Calculate median using the formula $N_{Me} = \frac{\sum f}{2}$

Test 4-10 (select a correct answer)

The first quartile is the value for which

1. The value for which 50% of the observations are smaller and 50% are larger

2. The value for which 25% of the observations are larger

3. The value for which 25% of the observations are smaller and 75% are larger

4. The value for which 20% of the observations are smaller and 80% are larger

PRACTICE

4.1. The salaries of 15 employees of a supermarket in April 2017 are the following:

Salaries of the employees of a supermarket

№	Name	Position	Salary, thousand rubles
1	Akimova N.	Seller	63,3
2	Begovaya L.	Manager	83,1
3	Belova B.	Accountant	78,3
4	Govorova I.	Seller	51,1
5	Denisov T.	Seller	62,6
6	Ivashov I.	Manager	77,3
7	Lenar A.	Seller	47,2
8	Motovidova S.	Manager	74,6
9	Orlova K.	Seller	63,3
10	Pravdina O.	Accountant	91,9
11	Presnyakova M.	Manager	77,9
12	Rostov J.	Manager	70,4
13	Rubleva V.	Seller	60,4
14	Sharova C.	Seller	52,3
15	Urchenko R.	Seller	63,3

Define:

1. Average salary.
2. Mode and median.
3. Quartiles.

4.2. The following is a set of ungrouped data ($n=9$):

20	16	12	8	17	9	8	19	10
----	----	----	---	----	---	---	----	----

Compute:

1. Mean, mode and median.
2. Quartiles and interquartile range.

4.3. Given the data on the length of service of males and females:

Length of service, year	Number of persons	
	male	female
1	6	2
2	10	6
3	15	7
4	20	10
5	12	14
6	10	18
7	8	16
8	6	9
9	2	6
10	6	2
Total:	95	90

Find for male and female:

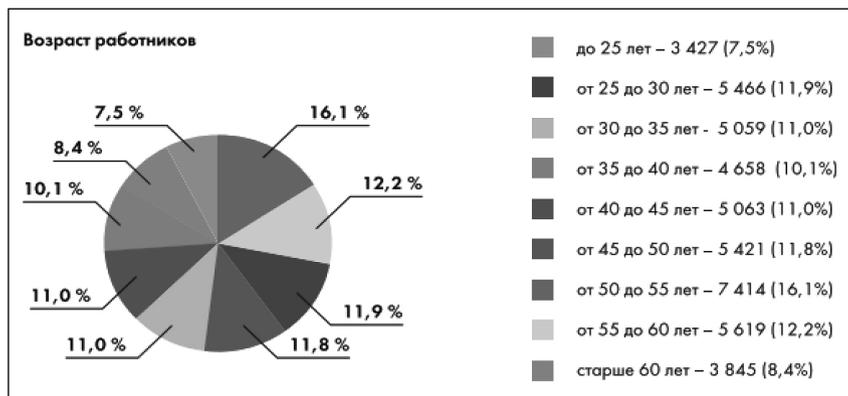
1. Mean.
2. Mode.
3. Median.
4. Quartiles.

4.4. Given the data on the distribution of employees by salary:

Salary, thousand rubles	Number of employees	
	Department 1	Department 2
under 40	30	10
40-50	35	20
50-60	55	30
60-70	70	70
70-80	120	85
80-90	90	130
90-100	60	100
100 and over	40	55
Total:	500	500

Determine the average, modal and median salary for each department.

4.5. There are data of the annual report of the Moscow metro on the employees' age, 01.01.2014:



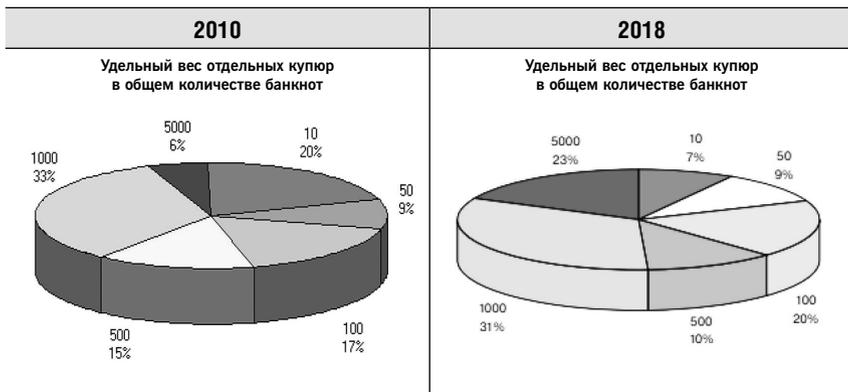
Source: Moscow metro state unitary enterprise "Annual report, 2014"

Perform the following tasks:

- Determine the average age of employees on the basis of:
 - absolute numbers of employees;
 - relative data (proportion) of the number of employees.
- Determine mode.

3. Determine median.
4. Determine quartiles.
5. Calculate the proportion of young people (employees under the age of 30 years) in the total number of employees.

4.6. There are data on the banknote proportion of money for 2010 and 2018 (%):



Source: Central Bank of the Russian Federation “Indicators of cash circulation» http://www.cbr.ru/Bank-notes_coins/nal/

Perform the following tasks:

- 1) Fill in the table according to the data presented on the chart:

Banknotes, rubles	The proportion of banknotes in the total number of banknotes, %	
	01.01.2010	01.01.2018
10		
50		
100		
500		
1000		
5000		

- 2) Determine the average nominal value of banknotes on 01.01.2010 and 01.01.2018.

4.7. There are Bank of Russia data on money transfers from the Russian Federation to the CIS countries, converted into us dollars:

Country	2010		2016	
	The average amount of one money translation's, dollars USA	The total amount of cash translation's, million dollars USA	The average amount of one money translation's, dollars USA	The total amount of cash translation's, million dollars USA
Azerbaijan	639	794	452	482
Armenia	632	1018	409	346
Belarus	295	165	237	115
Kazakhstan	522	247	719	327
Kyrgyzstan	369	1106	416	976
Moldova	516	845	348	252
Tajikistan	396	2216	200	579
Uzbekistan	584	2845	431	1888

Source: Bank of Russia " Cross-Border transfers of individuals (residents and non-residents)" <http://www.cbr.ru>

Compute the average amount of money transfers from Russia to CIS countries for 2010 and 2016.

4.8. Data on salary of employees:

Department	March		April	
	The average monthly salary, rubles	Number of employees	The average monthly salary, rubles	Wage fund, rubles
1	47 800	14	51 000	765 000
2	58 100	20	58 800	1 176 000

Determine the average monthly salary in March and in April.

4.9. There are data on exports of frozen and fresh fish:

	November 2016		December 2016	
	Quantity, thousand tons	Export revenue, million dollars USA	Price per ton, dollars. USA	Export revenue, million dollars USA
Foreign countries	76,7	127,7	1465,0	

Окончание

	November 2016		December 2016	
	Quantity, thousand tons	Export revenue, million dollars USA	Price per ton, dollars. USA	Export revenue, million dollars USA
CIS countries	3,8		1792,2	6,9
Total:		134,1		161,5

Define:

1. Average price per ton of fish in November and December.
2. The change of the average price of exported fish in December compared to November 2016.

4.10. There are data on the distribution of third-year students in their height:

Height, cm	Number of students
under 164	20
164 – 168	40
168 – 172	50
172 – 176	120
176 – 180	70
180 or over	30
Total:	330

Determine:

1. The average height using the arithmetic mean, mode and median.
2. The proportion of students with height 180 cm and above.

4.11. There are data on three branches of the organization:

Branch	Output in May, thousand rubles	Changes in output in May compared to April, %	Proportion of exported products in output in May, %
1	4540	101,5	25
2	5320	103,4	12
3	5900	105,8	15

Determine for the whole organization:

1. Percentage change of output in May compared to April.
2. Proportion of exported products in output in May.

4.12. There are data on the deputies' of the state Duma age:

State Duma of 2016-2021 convocation, persons

	Number	By the age, years				
		Under 30	30-40	40-50	50-60	60 and over
Deputies of the State Duma – total	449	3	53	129	142	122
including by fractions:						
– United Russia	343	–	37	98	117	91
– Communist party of RF	42	1	3	9	8	21
– Liberal–Democratic Party	39	2	13	18	4	2
– The Fair Russia	23	–	–	4	11	8

Source: Russian statistical yearbook. 2017

Determine:

1. Average age of deputies of each fraction.
2. The proportion of deputies under 30 years old for each fraction.

4.13. Given the distribution of population of Russia by per capita money income, based on data from household budget sample surveys and macroeconomic indicator average per capita money income of population, 2018:

Per capita money income, rubles per month	Proportion of population, %
under 7000	6
7000 – 10000	8
10000 – 14000	12
14000 – 19000	15
19000 – 27000	19
27000 – 45000	23
45000 – 60000	8
60000 and over	9
Total:	100,0

Define:

1. Median income per capita.
2. The 1-st and the 9-th deciles and the decile ratio.

4.14. Given the following distribution:

Annual turnover, million rubles	Up to 14	14 - 18	18 - 22	22 - 26	26 and more
Number of firms	3	7	12	8	20

Define:

1. Mean
2. Mode
3. Median
4. Quartiles

Make a conclusion

4.15. Given the following distribution:

Trade turnover, million rubles	Number of shops
50 - 60	15
60 - 70	7
70 - 80	6
80 - 90	4
90 - 100	8

Define:

1. Mean
2. Mode
3. Median

Make a conclusion

4.16. Given the following distribution:

Size of the family	2	3	4	5	6
Number of families, thousand	14	16	12	8	5

Define:

1. Mean
2. Mode
3. Median
4. Quartiles

Make a conclusion

4.17. Given the following distribution:

Age, years	Up to 20	20 - 30	30 - 40	40 - 50	50 and more
Number of man	6	12	22	34	26

Define quartiles and deciles.

Make a conclusion.

4.18. The following is the set of data from a sample of $n=12$:

7 3 8 2 10 2 4 9 2 5 6 5

Compute the mean, mode and median.

4.19. The following is the set of data from a sample of $n=15$:

7 3 8 2 10 2 4 9 2 5 6 5 2 9 1

Compute the mean, mode, median and quartiles.



Measures of Variation and Shape of a distribution

Learning objectives:

- Absolute measures of variation: range, mean linear deviation, variance, standard deviation, quartile deviation
- Relative measures of variation: coefficient of variation, coefficient of quartile deviation
- Extreme values (outliers)
- Types of variation
- Variation and standard deviation for the alternative variable
- Shape of a distribution

THEORY

Measures of variation give information on the spread, variability, dispersion of the data

Measures of Variance: main formulas

Indicator	Russian equivalent	Formula	
		For ungrouped data	For grouped data
<i>Absolute</i>			
Range	Размах вариации	$R = x_{\max} - x_{\min}$	

Indicator	Russian equivalent	Formula	
		For ungrouped data	For grouped data
Average linear deviation	Среднее линейное отклонение	$\bar{L} = \frac{\sum x - \bar{x} }{n}$	$\bar{L} = \frac{\sum x - \bar{x} \cdot f}{\sum f}$
Variance	Дисперсия	$\sigma^2 = \frac{\sum (x - \bar{x})^2}{n}$ Shortcuts: $\sigma^2 = \frac{\sum x^2}{n} - (\bar{x})^2$	$\sigma^2 = \frac{\sum (x - \bar{x})^2 \cdot f}{\sum f}$ Shortcuts: $\sigma^2 = \frac{\sum x^2 \cdot f}{\sum f} - (\bar{x})^2$
Standard deviation	Среднее квадратическое отклонение	$\sigma = \sqrt{\sigma^2}$	$\sigma = \sqrt{\sigma^2}$
Quartile deviation	Квартильное отклонение	$Q = \frac{Q_3 - Q_1}{2}$	$Q = \frac{Q_3 - Q_1}{2}$
Relative			
Coefficient of variation	Коэффициент вариации	$V = \frac{\sigma}{\bar{x}} \cdot 100\%$	$V = \frac{\sigma}{\bar{x}} \cdot 100\%$
Coefficient of Quartile Deviation	Коэффициент квартильного отклонения	$V_Q = \frac{Q_3 - Q_1}{2Q_2} \cdot 100\%$	$V_Q = \frac{Q_3 - Q_1}{2Q_2} \cdot 100\%$

Measures of variation: characteristics

1. The more data are spread out, the larger the measures of variation
2. The less data are spread out, the smaller the measures of variation
3. If there is no variation, all measures of variation will be zero

Coefficient of variation

Formula $V = \frac{\sigma}{\bar{x}} \cdot 100\%$

1. Measured as a percentage
2. Shows variation relative to mean
3. Can be used to compare the variability of several sets of data measured in different units
4. Measure the homogeneity of set of data
 - $V < 33\%$ – data set is homogeneous
 - $V > 33\%$ – data set is not homogeneous

Extreme values (outliers)

Formula

$$Z = \frac{x - \bar{x}}{\sigma}$$

x – data value

\bar{x} – mean

σ – standard deviation

A data value is considered an extreme outlier if its Z-score is less than -3.0 or greater than $+3.0$.

Types of variation

Variation	Russian equivalent	Formula
Total variation	Общая дисперсия	$\sigma_0^2 = \frac{\sum (x - \bar{x}_0)^2 \cdot f}{\sum f}$
Variation between groups	Межгрупповая дисперсия	$\delta^2 = \frac{\sum (\bar{x}_j - \bar{x}_0)^2 \cdot n_j}{\sum n_j}$
Average of the variation within groups	Средняя внутригрупповая дисперсия	$\bar{\sigma}_j^2 = \frac{\sigma_j^2 \cdot n_j}{\sum n_j}$ $\bar{\sigma}_i^2 = \frac{\sum (x_{ij} - \bar{x}_j)^2 \cdot f_{ij}}{\sum f_{ij}}$
The rule of summarizing of variation	Правило сложения дисперсий	$\sigma_0^2 = \delta^2 + \bar{\sigma}^2$

Determination Ratio

It is the ratio of the explained variation to the total variation

Formula $\eta^2 = \frac{\delta^2}{\sigma_0^2} \cdot 100\%$

Empirical Correlation Ratio

Formula $\eta = \sqrt{\frac{\delta^2}{\sigma_0^2}}$

Variation and standard deviation for the alternative variable

Alternative variable has only two values:

1 – there is a sign

0 – there is no a sign

p – proportion with code 1 (number of successes of total)

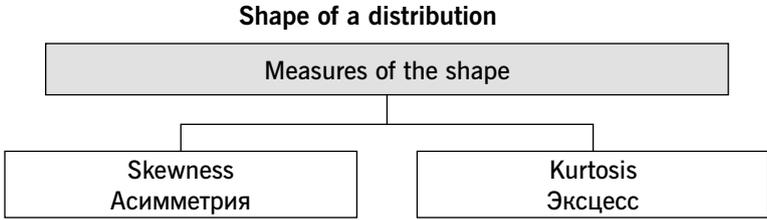
q – proportion with code 0

$$p + q = 1$$

$$\bar{x} = p$$

Variation: $\sigma^2 = p \cdot q$

Standard deviation: $\sigma = \sqrt{p \cdot q}$



Skewness (As)

Measures the extent to which data values are not symmetrical

Karl Pearson's formula

$$A_s = \frac{\bar{x} - M_o}{\sigma}$$

\bar{x} – mean

M_o – mode

σ – standard deviation

Value of skewness and shape of distribution

Left-Skewed distribution	Symmetric distribution	Right-Skewed distribution
$A_s < 0$	$A_s = 0$	$A_s > 0$
$\bar{x} < M_e$	$\bar{x} = M_e$	$\bar{x} > M_e$

$|A_s| < 0,25$

– small skewness

$0,25 < |A_s| < 0,50$

– moderate skewness

$|A_s| > 0,50$

– large skewness

Kurtosis (Ex)

Formula

$$E_x = \frac{\mu_4}{\sigma^4} - 3$$

$$\mu_4 = \frac{\sum (x - \bar{x})^4 \cdot f}{\sum f}$$

Value of kurtosis and shape of distribution

Plat distribution	Symmetric distribution	High peaked distribution
$E_x < 0$	$E_x = 0$	$E_x > 0$

TESTS

Test 5-1 (select a correct answer)

Range is the difference between

- 1) the difference between mean and minimum value

- 2) the difference between maximum value and mean
- 3) the difference between maximum and minimum values
- 4) sum of maximum and minimum values

Test 5-2 (select a correct answer)

Given the following data set:

3,0	4,5	6,0	6,5	7,4	7,5	8,0	8,7	9,0	10,0	101
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Researcher detected the technical error in the last observation and replaced 101 by 10,1. What happens to Quartile Deviation (QD) and Standard Deviation (SD)?

1. Both QD and SD will increase
2. QD will change but SD will stay the same
3. SD will decrease and QD will not change
4. Both QD and SD will decrease

Test 5-3 (select a correct answer)

Variance will be equal to 0 if all variables

1. Equal to 0
2. Equal to 100
3. Equal to each other
4. Have the same unit of measurement

Test 5-4 (select a correct answer)

The values of all variables are equal to 100. The variance is equal to

1. 100
2. 0
3. 10
4. 10000

Test 5-5 (select a correct answer)

The mean = 50, variance = 144. Coefficient of variation is equal to

1. 15
2. 24
3. 33
4. 45

Test 5-6 (select a correct answer)

Coefficient of variation = 64%, mean = 100. Variance is equal to

1. 4096
2. 5604
3. 1024
4. 2054

Test 5-7 (select a correct answer)

There are two data sets:

A: 10 17 20 30 15

B: 10 25 18 24 15

The coefficient of variation is

1. Larger in data set A
2. Larger in data set B
3. The same in data sets A and B
4. Equal to 0

Test 5-8 (select a correct answer)

The following is a set of data of $n = 5$:

6 15 5 9 2

Compute the Z scores. Are there any extreme values (outliers)?

1. No extreme value
2. 15
3. 2
4. 2 and 15

Test 5-9 (select a correct answer)

The following is a set of data of $n = 5$:

6 15 5 9 2

Are the data skewed?

1. Left-skewed
2. Right-skewed
3. Symmetric
4. Impossible to conclude

Test 5-10 (select correct answers)

Select absolute measures of variation

1. Standard deviation
2. Variance
3. Coefficient of variation
4. Range
5. Average linear deviation
6. Coefficient of quartile deviation

Test 5-11 (select correct answers)

Select relative measures of variation

1. Standard deviation
2. Variance
3. Coefficient of variation
4. Range
5. Average linear deviation
6. Coefficient of quartile deviation

Test 5-12 (select a correct answer)

Variation between groups = 81, average of variation within groups = 19. Determination ratio is equal

1. 81,0%
2. 23,5%
3. 90,0%
4. 43,6%

Test 5-13 (select a correct answer)

Skewness = 0,007. It means that distribution

1. Large right-skewed
2. Small right-skewed
3. Large left-skewed
4. Small left-skewed

Test 5-14 (select a correct answer)

Skewness = -0,890. It means that distribution

1. Large right-skewed
2. Small right-skewed
3. Large left-skewed
4. Small left-skewed

PRACTICE

5.1. There are data on prices of milk in 10 largest supermarkets:

Price, rubles per liter	70	72	75	73	76	72	74	75	74	73
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Define the measures of variance:

1. Range
2. Variance and standard deviation.
3. Coefficient of variation.

5.2. The following table gives the duration of tours to the Czech Republic:

Duration of tour, days	5	6	7	8	9
Proportion of tourists, %	5	25	40	20	10

Define the coefficient of variation

5.3. The following table gives the distribution of construction firms on the volume of investment:

Investment, million rubles	Number of firms
6–8	10
8–10	30
10–12	25
12–14	60
14–16	40
16–18	20
18–20	15
Total:	200

Define:

1. Coefficient of variation.
2. Skewness.

5.4. The investment attractiveness of the enterprises was estimated by points:

Points	under 0,8	0,8 – 1,0	1,0 – 1,2	1,2 and over	Total
Number of the enterprises	3	6	9	2	20

Define mean linear deviation and standard deviation.

5.5. Average monthly expenses of households in 2011 were the following:

1. on food – 8700 rubles, $\sigma = 1392$ rubles;
2. on nonfoods – 10200 rubles, $\sigma = 6120$ rubles.

Compare the variation of expenses of households on food and on nonfoods.

5.6. The following table gives the distribution of employees by the length of service:

Length of service, years	Employees, persons	
	Department 1	Department 2
0 – 5	2	7
5 – 10	15	23

Окончание

Length of service, years	Employees, persons	
	Department 1	Department 2
10 – 15	20	12
15 – 20	3	8

Which one of the distributions of employees is more homogeneous?

5.7. The following results have been received by the sociometric research of professional level of supermarket's employees (in points):

Measure	Sellers	Warehouse workers
Mode	40,0	60,0
Mean	42,7	63,6
Variance	36,0	64,0

Compare the skewness of these distributions.

5.8. There are following data on distribution of insurance companies by the size of authorized capital, million rubles:

Authorized capital, million rubles	under 6	6-8	8-10	10 and over
Number of insurance companies	28	37	20	15

Define skewness using Karl Pearson's formula.

5.9. The following table gives the distribution of students by the costs of a lunch:

Costs of a lunch, rubles	Number of students
110 – 124	3
124 – 138	50
138 – 152	100
152 – 166	40
166 – 180	4
180 and more	3

Define standard deviation and coefficient of variation.

5.10. Define the mean if the coefficient of variation 30%, and variance is equal to 900.

5.11. The mean is equal to 2600 and the coefficient of variation is equal to 30%.

Define variance.

5.12. Given the following data on education and wages:

Education	Number of persons	The average wage, thousand rubles	Variation within groups
Technical	32	30	50
Economic	68	50	60

Define:

1. Variation between groups.
2. Average of the variation within groups.
3. Total variation.
4. Determination ratio.
5. Empirical correlation ratio

5.13. The data registered by the employment service indicate that there is a relationship between the level of education of the unemployed and the period of work interruption:

Level of education	Number of unemployed, persons	Average period of work interruption, month
Secondary	70	3
Secondary vocational	85	6
Higher	95	9

Total variation equal to 7,8.

Define:

1. Variation between groups.
2. Determination ratio.

5.14. Given data of development of mobile asses to the internet in countries with different level of income:

Level of income	Number of countries	The average number of Internet users per 1,000 people	Variation within group
Low	24	15	22000
Middle	66	70	38000
High	30	470	50000

Define:

1. Variation between groups.
2. Average of the variation within groups.
3. Total variation.
4. Determination ratio.

5.15. 10 of the 200 products in the storage were defective.

Determine the variance and the standard deviation.

5.16. According to the sample survey of the Federal state statistics service "Influence of behavioral factors on the health status of the population" it was found out that 64% of respondents do not comply with the food regime.

Determine the standard deviation of the proportion of the population that does not comply with the diet.

5.17. Given the number of inhabitants of Central Federal District of Russian Federation on January 1, 2017:

№	Region	Number of inhabitants per 1 sq. km
1	Belgorod Region	57
2	Bryansk Region	35
3	Vladimir Region	48
4	Voronezh Region	45
5	Ivanovo Region	48
6	Kaluga Region	34
7	Kostroma Region	11
8	Kursk Region	37
9	Lipetsk Region	48
10	Moscow Region	168

№	Region	Number of inhabitants per 1 sq. km
11	Orel Region	31
12	Ryazan Region	28
13	Smolensk Region	19
14	Tambov Region	30
15	Tver Region	15
16	Tula Region	58
17	Yaroslavl Region	35
18	Moscow city	4762

Source: Russian statistical yearbook. 2017

Perform the following tasks:

1. Compute mean, standard deviation and coefficient of variation.
2. Compute Z scores and find regions with extreme values of number of inhabitants per 1 sq. km.
3. Remove regions with extreme values from the set of data and compute mean, standard deviation and coefficient of variation again. Do this until the population becomes homogeneous. Call the regions with extreme values of number of inhabitants per 1 sq. km.

5.18. Given the data on the construction companies of the region on the ownership type and number of employees:

№	Ownership type	Number of employees	№	Ownership type	Number of employees
1	state and municipal	90	15	private	68
2	private	170	16	state and municipal	80
3	state and municipal	80	17	private	50
4	private	110	18	other	80
5	private	160	19	private	104
6	other	100	20	private	40
7	private	63	21	private	55
8	private	75	22	state and municipal	94
9	private	72	23	private	154

Окончание

№	Ownership type	Number of employees	№	Ownership type	Number of employees
10	private	80	24	private	63
11	private	65	25	private	73
12	other	90	26	private	110
13	private	72	27	private	88
14	state and municipal	60	28	state and municipal	70

Perform the following tasks:

1. Group the companies by ownership type.
2. Determine the coefficient of variation for each group and compare the groups by homogeneity of distribution companies of the region by the number of employees.

5.19. There are data on the deputies' of the state Duma age:

State Duma of 2016–2021 convocation

persons

	Number	By the age, years				
		Under 30	30-40	40-50	50-60	60 and over
Deputies of the State Duma – total	449	3	53	129	142	122
including by fractions:						
– United Russia	343	–	37	98	117	91
– Communist party of RF	42	–	3	9	8	21
– Liberal-Democratic Party	39	2	13	18	4	2
– The Fair Russia	23	–	–	4	11	8

Source: Russian statistical yearbook. 2017

Determine:

1. Coefficient of variation for each fraction.
2. Compare the fractions by homogeneity of distribution deputies by the age.

5.20. Given the data on wages of supermarket employees:

Profession	Number of employees	Wage, thousand rubles				
Seller	5	45,7	45,9	46,0	45,8	45,1
Consultant	2	47,3	46,9			
Cashier	3	49,0	48,9	51,2		

Calculate:

1. Variance for each group (to the nearest 0,001)
2. Average of the variation within groups.
3. Variation between groups.
4. Total variation.
5. Determination ratio.
6. Empirical correlation ratio

5.21. Given the following distribution:

Annual turnover, million rubles	Up to 14	14 - 18	18 - 22	22 - 26	26 and more
Number of firms	3	7	12	8	20

Define:

1. Variance.
2. Standard deviation.
3. Coefficient of variation.
4. Skewness.
5. Make a conclusion

5.22. Given the following distribution:

Bank deposit, thousand rubles	Up to 40	40 - 60	60 - 80	80 - 100	100 and more
Number of the depositors	120	280	100	80	40

Define skewness and make a conclusion about the form of the distribution.

5.23. The data in the file contain the bounced check fees, in dollars, for a sample of 24 banks for direct-deposit customers who maintain a \$100 balance:

27 29 21 21 22 23 26 26 19 26 16 20
19 21 26 26 23 30 30 30 16 21 30 25

Perform the following tasks:

1. Compute the variance, standard deviation, range interquartile range, coefficient of variation, and Z scores.
2. Are the data skewed?

5.24. The following is the set of data from a sample of $n=15$:

7 3 8 2 10 2 4 9 2 5 6 5 2 9 1

Compute the variance, standard deviation, range interquartile range, coefficient of variation, and Z scores.

UNIT 6



Sample method

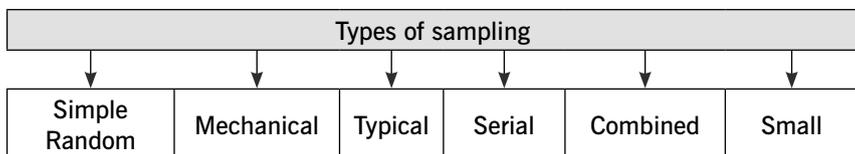
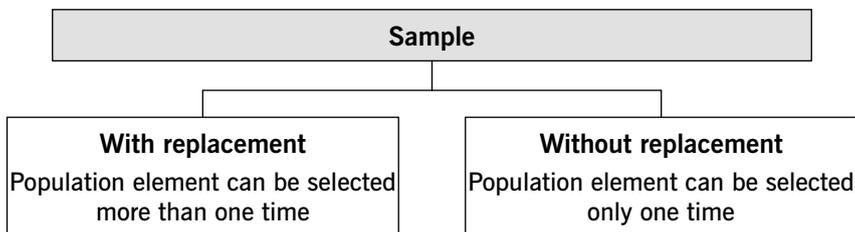
Learning objectives:

- Population and sample parameters
- Sample with replacement and without replacement
- Confidence intervals for the mean and proportion
- Determining sample size
- Determining the probability

THEORY

Basic definitions

Definition	Russian equivalent	Characteristic
Population	генеральная совокупность	includes all elements or individuals of set of data
Sample	выборочная совокупность	is a portion of population selected for analysis
Point estimates	точечная оценка	sample statistics
Margin error	предельная ошибка	distance between the point estimate and the population parameter
Confidence interval	доверительный интервал	an interval estimate for a population parameter with the level of confidence



Population and Sample Parameters

Parameter	Population	Sample
Size	N	n
Mean	\bar{x}	\tilde{x}
Variance	σ^2	S^2
Standard deviation	σ	S
Proportion	p	w

1. Determination of confidence interval

Confidence interval for mean

$$\tilde{x} - \Delta_{\tilde{x}} \leq \bar{x} \leq \tilde{x} + \Delta_{\tilde{x}}$$

Margin error

$$\Delta_{\tilde{x}} \leq t \cdot \mu_{\tilde{x}}$$

t – critical value depended on the confidence level

$\mu_{\tilde{x}}$ – standard error of mean

Formulas for standard error of mean

$\mu_{\tilde{x}}$	Sample	
	with replacement	without replacement
Standard error of the mean	$\mu_{\tilde{x}} = \sqrt{\frac{S^2}{n}}$	$\mu_{\tilde{x}} = \sqrt{\frac{S^2}{n} \left(1 - \frac{n}{N}\right)}$

Confidence interval for proportion

$$w - \Delta_w \leq p \leq w + \Delta_w$$

Margin error

$$\Delta_w \leq t \cdot \mu_w$$

t – critical value depended on the confidence level

μ_w – standard error of proportion

Formulas for standard error of proportion

μ_w	Sample	
	with replacement	without replacement
Standard error of proportion	$\mu_w = \sqrt{\frac{w(1-w)}{n}}$	$\mu_w = \sqrt{\frac{w(1-w)}{n} \cdot \left(1 - \frac{n}{N}\right)}$

Common confidence levels and the t critical value

P	t
0,683	1,00
0,866	1,50
0,95	1,96
0,954	2,00
0,99	2,58
0,997	3,00
0,999	4,00

2. Determining the sample size n

Formulas for sample size

Sample size (n)	Sample	
	with replacement	without replacement
For the mean	$n = \frac{t^2 \cdot S^2}{\Delta_x^2}$	$n = \frac{t^2 \cdot N \cdot S^2}{\Delta_x^2 \cdot N + t^2 \cdot S^2}$
For proportion	$n = \frac{t^2 \cdot w(1-w)}{\Delta_w^2}$	$n = \frac{t^2 \cdot N \cdot w(1-w)}{\Delta_w^2 \cdot N + t^2 \cdot w(1-w)}$

3. Determining the probability P

Steps:

1) Determining critical value t

For the mean	$t = \frac{\Delta_{\bar{x}}}{\mu_{\bar{x}}}$
For proportion	$t = \frac{\Delta_w}{\mu_w}$

2) Determining the probability P corresponding to critical value t .

TESTS

Test 6-1 (select a correct answer)

Sample is

1. All elements or individuals of set of data
2. The largest units of the population
3. A portion of population selected for analysis
4. The smallest units of the population

Test 6-2 (select an incorrect answer)

Types of sampling:

1. Serial
2. Alternative
3. Typical
4. Simple random
5. Small
6. Combined

Test 6-3 (select a correct answer)

Determine the missing indicator in the formula for sample size without replacement:

$$n = \frac{t^2 \cdot N \cdot S^2}{\dots \cdot N + t^2 \cdot S^2}$$

1. \bar{x}
2. n
3. $\mu_{\bar{x}}$
4. $\Delta_{\bar{x}}^2$

Test 6-4 (select a correct answer)

Choose the correct formula for standard error of proportion with replacement:

$$1. \mu_w = \sqrt{\frac{w(1-w)}{n} \cdot \left(1 - \frac{n}{N}\right)}$$

$$2. \mu_w = \sqrt{\frac{w(1-w)}{n}}$$

$$3. \mu_{\bar{x}} = \sqrt{\frac{S^2}{n} \left(1 - \frac{n}{N}\right)}$$

$$4. \mu_{\bar{x}} = \sqrt{\frac{S^2}{n}}$$

Test 6-5 (select correct answers)

Standard error of the mean depends of:

1. critical value
2. population size
3. sample size
4. population standard deviation
5. sample standard deviation

Test 6-6 (select a correct answer)

How many times will change the standard error if the sample size is increased by 4 times?

1. Reduced by 2 times
2. Increased by 2 times
3. Reduced by 4 times
4. Increased by 4 times

Test 6-7 (select a correct answer)

How many times will change the sample size if the sample variance is increased by 2 times?

1. Reduced by 2 times
2. Increased by 2 times
3. Reduced by 4 times
4. Increased by 4 times

Test 6-8 (select a correct answer)

A random sampling is carried out in the bank to determine the average size of the deposit. Standard error = 100, variance = 2500, probability = 0,683 ($t=1$). Calculate sample size.

- | | | |
|--------|---------|---------|
| 1. 25 | 3. 1000 | 5. 2500 |
| 2. 250 | 4. 2000 | |

Test 6-9 (select a correct answer)

Random sampling with replacement was organized. Among 400 students, 40 people were unsuccessful in the exam. Determine the maximum percent of unsuccessful students with probability 0,954 ($t=2$):

- | | |
|-------|-------|
| 1. 10 | 3. 15 |
| 2. 13 | 4. 20 |

Test 6-10 (select a correct answer)

Random sampling with replacement was organized. Among 400 students, 40 people were unsuccessful in the exam. Determine the minimum percent of unsuccessful students with probability 0,954 ($t=2$):

- | | |
|-------|------|
| 1. 20 | 3. 7 |
| 2. 10 | 4. 4 |

PRACTICE

6.1. Simple random sample with replacement was organized in one of the supermarkets. 200 buyers have been interviewed about purchase cost. The following data were received:

Purchase cost, thousand rubles	Under 2	2 - 4	4 - 6	6 - 8	8 - 10
Number of buyers	80	40	50	10	20

With the probability 0,954 (95,4%) *define*:

- Confidence interval for average purchase cost.
- Confidence interval for proportion of purchase cost 8 thousand rubles and more.

6.2. To develop measures for prevention of delay of employees for the office the 30% simple random sample without replacement was organized. The following data are obtained:

Time for the road from home to the office, min	Under 30	30 - 40	40 - 50	50 - 60	60 and over
Number of employees, persons	70	80	200	55	45

Define:

- With probability 0,997 (99,7%) confidence interval for average time for the road from home to the office.
- With probability 0,954 (95,4%) confidence interval for proportion of employees spending for the road from home to the office 1 hour and more.

6.3. 10% simple random sample without replacement was organized in one of the firms. 70 employees have been interviewed whether they are satisfied with their salary.

20% of interviewed employees were satisfied with the salary.

With probability 0,954 (95,4%) *define* confidence interval for proportion of employees satisfied with their salary.

6.4. In 2017 there were 500 gas stations in the region. The service of environmental control has carried out 10% random inspection of gas stations. It has been revealed that 6 gas stations sold counterfeit gasoline.

With probability 0,95 (95%) *define* confidence interval for proportion of gas stations that may sell counterfeit gasoline.

6.5. There are 73 thousand people in the region. It is planned to survey the population to determine the average cost of medical services and medicines. A trial survey showed that standard deviation is equal to 38 rubles.

With the probability 0,954 *define* the size of simple random sample without replacement that the margin error hasn't exceeded 10 rubles.

6.6. Last year the tax inspection has revealed 32% of the firms detaining tax payments. How many firms does the tax inspection need to check this year that the margin error hasn't exceeded 5%? The probability is equal to 0,954.

6.7. Given the results of the 10% simple random sample without replacement of 250 families:

The total area of residential premises, falling on average on one inhabitant, m²	Number of households
Under 21	19
21 – 23	21
23 – 25	46
25 – 27	54
27 – 29	44
29 – 31	30
31 – 33	14
33 – 35	12
35 and over	10
Total:	250

Define:

1. Sample average total area of residential premises per one person.
 2. Sample variance.
 3. Standard error of mean.
 4. Coefficient of variation.
 5. Mode and median.
 6. Margin error of mean with probability 0,997.
 7. Confidence interval for average total area of residential premises per one person.
 8. Margin error of proportion of households with the total area of residential premises per one person up to 31 м² with probability 0,954
 9. Confidence interval for proportion of households with the total area of residential premises per one person up to 31 м² with probability 0,954.
- Comment on the results.

6.8. Given the results of the 8% simple random sample without replacement of 800 households:

Share of food purchases in total household expenditure, %	Number of households, %
Under 50	2
50 – 54	5
54 – 58	9
58 – 62	16
62 – 66	20
66 – 70	18
70 – 74	16
74 and over	14
Total:	100

Define:

1. Sample average share of food purchases in total household expenditure.
2. Sample variance.
3. Standard error of mean.
4. Coefficient of variation.
5. Mode and median.
6. Margin error of mean with probability 0,997.
7. Confidence interval for average share of food purchases in total household expenditure.

8. Confidence interval for proportion of households with share of food purchases in total household expenditure not less than 58% and not more than 70% with probability 0,954.

Comment on the results.

6.9. A sample survey of voters was conducted in the region to study the popularity of candidate "A". The survey covered 400 people (selected by the method of mechanical selection). In total 25000 voters are registered in the region. 320 respondents are going to support the candidate "A".

Define:

- 1) the proportion of people voting for candidate "A", based on the results of the sample survey;
- 2) standard error of proportion;
- 3) with probability 0,954 margin error of proportion
- 4) confidence interval for proportion.

6.10. Mosgorstat plans to conduct a sample survey of public catering enterprises. The purpose of the survey – the profitability of companies. At the time of the survey 2480 companies were registered in the city. The error should not exceed 1% with a probability of 0.997. Standard deviation should not exceed 2,8%.

Define the size of simple random sample:

- a) without replacement;
- b) with replacement.

6.11. How the margin error will change if the variance reduced by 4 times, size of simple random sample will increase by 9 times, and the confidence probability will increase from 0,683 to 0,997?

6.12. According to the sample survey (with replacement), the variance of the average term of using a short – term loan to the first branch of the Bank was 256, and in the second branch – 100. The number of surveyed credit accounts of the first branch is 4 times more than the second. In what branch average sampling error more?



Correlation and regression analysis

Learning objectives:

- Types of correlation
- Scatterplot
- Correlation coefficient
- Correlation ratio
- Equation of the least-squares regression line
- Nonparametric methods
- Multiple correlation

THEORY

Correlation analysis is used to measure the strength of relationship between variables.

Regression analysis is used to predict variable based on the value of at least one independent.

Types of variables

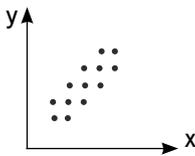
Type of variable	Russian equivalent	Characteristic
Dependent (Y)	зависимая переменная	the variable we wish to predict or explain
Independent (X)	независимая переменная	the variable used to predict or explain the dependent variable

Types of correlation

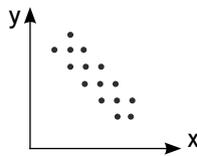
DIRECTION	<ul style="list-style-type: none"> • Positive / Direct (the dependent variable Y increases as the independent variable X tends to increase) • Negative / Inverse (the dependent variable Y decreases as the independent variable X tends to increase)
FORM	<ul style="list-style-type: none"> • Linear (the equation of the line) • Non-Linear (the equation of a parabola, hyperbola and so on)
NUMBER OF VARIABLES	<ul style="list-style-type: none"> • Partial (relationship between a dependent and an independent variables) • Total (relationship between several dependent and one independent variables)

A scatter plot can be used to show the relationship between two variables. The data is plotted as points on a graph where the independent variable is the horizontal axis and the dependent variable is the vertical axis. The pattern formed by the points on a scatterplot indicates the strength of the relationship between the two variables.

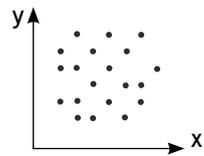
If points are randomly scattered there is no correlation, no relationship between two variables.



Positive correlation



Negative correlation



No correlation

Method of analytical grouping

1. All observations are divided into groups by the value of the independent variable (X)
2. Average values of dependent variable are calculated (\bar{Y}) for each group.

Correlation coefficient

Correlation coefficient measures the strength and direction of the relationship between two quantitative variables.

Karl Pearson's coefficient of correlation	Karl Pearson's formula for short-cut method
$r = \frac{\sum xy - \frac{\sum x \cdot \sum y}{n}}{\sqrt{(\sum x^2 - \frac{(\sum x)^2}{n}) \cdot (\sum y^2 - \frac{(\sum y)^2}{n})}}$	$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$ $r = \frac{\bar{xy} - \bar{x} \cdot \bar{y}}{\sigma_x \cdot \sigma_y}$

For the Pearson r correlation, both variables should be normally distributed

Interpretation of correlation coefficient value

$$-1 \leq r \leq +1$$

$r < 0$ – negative correlation

$r > 0$ – positive correlation

$r = 0$ – no linear correlation

r - value	Relationship
up to $\pm 0,3$	no linear relationship
$\pm 0,3 - \pm 0,5$	weak
$\pm 0,5 - \pm 0,7$	moderate
$\pm 0,7 - \pm 0,9$	strong
$\pm 0,9$ and more	very strong

Correlation ratio

$$\eta = \sqrt{\frac{\delta^2}{\sigma_0^2}} \quad 0 \leq \eta \leq +1$$

Types of variation

Variation	Russian equivalent	Formula
Total variation	Общая дисперсия	$\sigma_0^2 = \frac{\sum (y - \bar{y}_0)^2 \cdot f}{\sum f}$
Variation between groups	Межгрупповая дисперсия	$\delta^2 = \frac{\sum (y_i - \bar{y}_0)^2 \cdot n_i}{\sum n_i}$
Average of the variation within groups	Средняя внутригрупповая дисперсия	$\bar{\sigma}_j^2 = \frac{\sum \sigma_j^2 \cdot n_j}{\sum n_j}$

Variation	Russian equivalent	Formula
The rule of summarizing of variation	Правило сложения дисперсий	$\sigma_0^2 = \delta^2 + \bar{\sigma}^2$

The equation of the least-squares regression line

$$\hat{y} = a_0 + a_1 \cdot x$$

The system of equations for determining the parameters of the equation

$$\begin{cases} na_0 + a_1 \sum x = \sum y \\ a_0 \sum x + a_1 \sum x^2 = \sum xy \end{cases}$$

Formulas for parameters a and b:

Parameter a (intercept)	Parameter b (coefficient of regression)
$a_0 = \bar{y} + a_1 \bar{x}$	$a_1 = \frac{\sum xy - n \cdot \bar{x} \cdot \bar{y}}{\sum x^2 - n \cdot (\bar{x})^2}$ or $a_1 = r \cdot \frac{\sigma_y}{\sigma_x}$

Nonparametric methods



Between quantitative variables
Spearman rank correlation coefficient
Kendall rank correlation coefficient

Between qualitative variables
Coefficient of association
Contingency coefficient

Spearman rank correlation coefficient

$\rho = 1 - \frac{6 \cdot \sum_{i=1}^n d_i^2}{n \cdot (n^2 - 1)}$	d_i – differences between paired ranks d_i^2 – squared differences between paired ranks n – number of pairs (number of observations)
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$$-1 \leq \rho \leq +1$$

- $\rho < 0$ – negative correlation
- $\rho > 0$ – positive correlation
- $|\rho| > 0,5$ – relationship is significant

Steps of calculating Spearman rank correlation coefficient

1. Rank x-values from lowest to highest
2. Rank y-values from lowest to highest

3. Compute differences between paired ranks
4. Compute squared differences between paired ranks
5. Compute Spearman rank coefficient
6. Interpret the value of Spearman correlation coefficient
7. Kendall rank correlation coefficient

$\tau = \frac{2 \cdot S}{n \cdot (n - 1)}$	n – number of observations $S = P + Q$ P – the number of ranks following the current one and exceeding its value Q – the number of ranks following the current one and less than its value
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$$-1 \leq \tau \leq +1$$

$\tau < 0$ – negative correlation

$\tau > 0$ – positive correlation

$|\tau| > 0,5$ – relationship is significant

Steps of calculating Kendall rank correlation coefficient

1. Rank x-values from lowest to highest
2. Rank y-values from lowest to highest
3. Determine for each rank y the number of ranks following it that exceed its value. The resulting numbers are summed and denoted by P, which is taken with the sign “+”
4. Determine for each rank y the number of subsequent ranks less than its value. The resulting numbers are summed and denoted by Q, which is taken with the sign “-“
5. Determine $S = P + Q$
6. Compute Kendall rank coefficient
7. Interpret the value of Kendall rank correlation coefficient.

Relationship between qualitative variables

Coefficient of association and contingency coefficient

2x2 table

Variable x	Variable y		Total:
	y1	y2	
x1	a	b	a+b
x2	c	d	c+d
Total:	a+c	b+d	n

Coefficient of association	Contingency coefficient
$K_a = \frac{ad - bc}{ad + bc}$ $-1 \leq K_a \leq +1$ $ K_a > 0,5 - \text{relationship is significant}$	$K_c = \frac{ad - bc}{\sqrt{(a+b) \cdot (b+d) \cdot (a+c) \cdot (c+d)}}$ $-1 \leq K_c \leq +1$ $ K_c > 0,3 - \text{relationship is significant}$

Multiple regression

Examine the liner relationship between 1 dependent (Y) and two or more independent (Xi) variables.

Linear equation of multiple regression:

$$\hat{y}_{1,2,\dots,k} = a_0 + a_1x_1 + a_2x_2 + \dots + a_kx_k$$

a_1, a_2, \dots, a_k – coefficients of regression

x_1, x_2, \dots, x_k – independent variables

$\hat{y}_{1,2,\dots,k}$ – theoretical values of dependent variable

The system of equations for determining the parameters of the equation of multiple regression (for two independent variables):

$$\begin{cases} na_0 + a_1\sum x_1 + a_2\sum x_2 = \sum y \\ a_0\sum x_1 + a_1\sum x_1^2 + a_2\sum x_1x_2 = \sum x_1y \\ a_0\sum x_2 + a_1\sum x_1x_2 + a_2\sum x_2^2 = \sum x_2y \end{cases}$$

For the analysis of influence factors on the dependent variable we determine the coefficients of elasticity and β -coefficients.

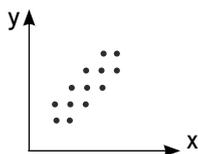
Coefficient of elasticity and β -coefficient

Coefficient	Russian equivalent	Formula	Characteristics
Coefficient of elasticity (\mathcal{E}_j)	Коэффициент эластичности	$\mathcal{E}_j = b_j \cdot \frac{\bar{x}_j}{\bar{y}}$	Shows percent of the average change of the dependent variable due to change of the independent variable by 1%
β -coefficient (β_j)	Бета-коэффициент	$\beta_j = b_j \cdot \frac{\sigma_{x_j}}{\sigma_y}$	Shows the change of standard deviation of the dependent variable due to change of the independent variable by the value of standard deviation

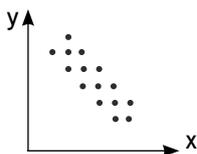
TESTS

Test 7-1 (select a correct answer)

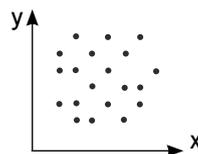
Determine the correspondence between graphs and the direction of correlation:



a



b



c

1. no correlation
2. positive correlation
3. negative correlation

Test 7-2 (select a correct answer)

The value of linear correlation coefficient equal to 0 indicates. It means:

1. Functional relationship between variables
2. Positive relationship between variables
3. Negative relationship between variables
4. No relationship between variables

Test 7-3 (select a correct answer)

The linear correlation coefficient can take values

1. From -1 to 0
2. From 0 to +1
3. From -1 to +1
4. Any value

Test 7-4 (select a correct answer)

There is a close negative linear relationship between dependent and independent variables. Proof of this is:

1. Correlation ratio = 0,7
2. Linear correlation coefficient = 0,8
3. Coefficient of regression = -1,5
4. Linear correlation coefficient = -0,8

Test 7-5 (select a correct answer)

The value of the Spearman rank correlation coefficient equal to -0.740 indicates that

1. No relationship between variables
2. Close negative relationship
3. Weak negative relationship
4. Close positive relationship

Test 7-6 (select a correct answer)

Correlation ratio is equal to $0,80$. Define how many percent of the variation in y can be explained by the variation in x

- | | |
|--------|--------|
| 1. 36% | 3. 20% |
| 2. 80% | 4. 64% |

Test 7-7 (select a correct answer)

Correlation ratio is equal to $0,9$. Total variation is equal to $7,4$. Define the value of variance between groups

- | | |
|----------|----------|
| 1. $6,0$ | 3. $1,4$ |
| 2. $8,8$ | 4. $6,5$ |

Test 7-8 (select correct answers)

Choose the measures of the strength of relationship between qualitative variables:

1. Coefficient of association
2. Correlation coefficient
3. Kendall rank correlation coefficient
4. Correlation ratio
5. Spearman rank correlation coefficient
6. Contingency coefficient

Test 7-9 (select correct answers)

Choose the measures of the strength of relationship between quantitative variables:

1. Coefficient of association
2. Correlation coefficient
3. Kendall rank correlation coefficient
4. Correlation ratio
5. Spearman rank correlation coefficient
6. Contingency coefficient

Test 7-10 (select a correct answer)

The correlation ratio can take values

1. From -1 to 0
2. From 0 to +1
3. From -1 to +1
4. Any value

PRACTICE

7.1. There are the following data:

Number of the shop	Advertising costs, million rubles (X)	Trade turnover, million rubles (Y)
1	0,2	4
2	0,3	5
3	0,4	6
4	0,4	7
5	0,5	6
6	0,6	7
7	0,6	7
8	0,7	6
9	0,8	8
10	0,9	8

Perform the following tasks:

1. Construct a scatterplot.
2. Define Karl Pearson's coefficient of correlation.
3. Determine the equation of regression.
4. Determine the predicted y-value for $x=1,0$ million rubles.

7.2. Given the data on length of service and the average monthly wage of 10 employees of the firm:

N	Length of service, years	Average monthly wage, thousand rubles
1	2	21
2	2	22

Окончание

N	Length of service, years	Average monthly wage, thousand rubles
3	3	26
4	4	30
5	5	33
6	5	30
7	5	37
8	6	36
9	6	39
10	7	39

Perform the following tasks:

1. Define Karl Pearson's coefficient of correlation.
2. Make a conclusion about relationship between length of service and average monthly wage.

7.3. There are the following data:

Percentage of defective products, %	Length of service, years
1,2	3
1,4	4
1,5	5
1,9	5
2,0	4
2,1	7
2,2	8
2,5	9
2,7	12

Perform the following tasks:

1. Determine dependent and independent variables.
2. Construct a scatterplot.
3. Define Karl Pearson's coefficient of correlation.
4. Determine the equation of regression.
5. Determine the predicted y-value for $x=10$ years.

7.4. The data registered by the employment service indicate that there is a relationship between the level of education of the unemployed and the period of work interruption:

Level of education	Number of unemployed, persons	Average period of work interruption, month
Secondary	70	3
Secondary vocational	85	6
Higher	95	9

Total variation equal to 7,8.

Define the correlation ratio.

7.5. Given data of development of mobile asses to the internet in countries with different level of income:

Level of income	Number of countries	The average number of Internet users per 1,000 people	Variation within group
Low	24	15	22000
Middle	66	70	38000
High	30	470	50000

Define the correlation ratio.

7.6. Correlation ratio is equal to 0,8. The total variation is equal to 10.

Define the variation between groups.

7.7. Given the results of the survey of second-year students:

Learning achievement	Number of students		Total:
	go in for sports	not go in for sorts	
Good	220	60	280
Bad	10	30	40
Total:	230	90	320

Define coefficient of association and contingency coefficient.

7.8. Given the result of the survey of 500 children the color of hair and eyes of every child was determined:

Eye color	Hair color	
	blond	dark
Light	25	50
Dark	150	275

Determine the coefficient of association and *define*, if there is a relationship between the color of hair and the color of eyes.

7.9. Given the ranks of prices of supply and demand for shares of the largest enterprises:

№ of enterprise	Rank	
	Price of supply (X)	Price of demand (Y)
1	5	4
2	1	3
3	6	5
4	3	2
5	2	1
6	4	6
7	7	7

Perform the following tasks:

1. Define Spearman rank correlation coefficient.
2. Define Kendall rank coefficient.
3. Make a conclusion about the relationship between the price of supply and the price of demand.

7.10. Given the following data for the banks:

Number of the bank	Assets, million rubles	Profit, million rubles
1	866	40
2	328	18
3	207	13
4	185	15
5	109	4
6	104	16

Окончание

Number of the bank	Assets, million rubles	Profit, million rubles
7	327	6
8	113	10
9	91	3
10	849	14

Define Spearman rank correlation coefficient and make a conclusion about the relationship between the assets and the profit.

7.11. There are data on the quality of roads and the number of road accidents:

Region	The proportion of good quality roads, %	Number of road accidents, per 100 thousand people
Belgorod	87,6	80
Bryansk	34,8	144
Vladimir	25,3	196
Voronezh	43,1	139
Ivanovo	80,4	165
Kaluga	41,0	204
Kostroma	41,1	116
Kursk	42,2	166
Lipetsk	52,7	150
Moscow	54,4	108

Define Spearman rank correlation coefficient and make a conclusion about the relationship between the proportion of good quality roads and number of road accidents.

7.12. For a group of commercial banks an equation was calculated that reflects the dependence of the size of profit on the size of assets (x):

$$\hat{y} = 0,50 + 1,30 x$$

Additional information:

60 banks, average assets -700 million rubles, and the average profit -350 million rubles.

Determine the percentage of change of profit with an increase assets by 1%.

7.13. According to the analysis of the influence of factors on the average wages of employees of 50 enterprises in January 2019, the following regression equation is obtained: $\hat{y} = 5100 + 50x_1 + 710x_2 - 30x_3$

Additional information:

Variable	Unit of measure	Mean	Standard deviation
	%	70,0	10,0
	%	2,9	0,5
	%	10,0	2,0
	rub	30500	2500

Determine which of the factors has the greatest impact on the average wage of workers. Formulate a conclusion.

7.14. Given the 15% simple random sample:

N _e of enterprise	Cost of sales, thousand rubles per piece X	Production costs, thousand rubles Y	N _e of enterprise	Cost of sales, thousand rubles per piece X	Production costs, thousand rubles Y
1	1221	371	16	1251	370
2	1581	627	17	1588	640
3	1219	350	18	1354	425
4	1243	351	19	1649	645
5	1402	412	20	1559	565
6	1473	445	21	1191	253
7	1448	478	22	1195	340
8	1465	452	23	1284	356
9	1072	258	24	1182	407
10	1265	349	25	1326	418
11	1191	261	26	1385	435
12	1374	435	27	1177	405
13	1514	536	28	1388	468
14	1228	302	29	1404	434
15	1474	525	30	1498	558

Perform the following tasks:

1. Check the possibility of using the correlation method:
 - define mean, standard deviation and coefficient of variation of x-value.
2. Define existence and direction of relationship between X and Y using:
 - scatterplot;
 - method of analytical group.
3. Estimate the strength of the relationship counting:
 - Karl Pearson's coefficient of correlation;
 - correlation ratio.
4. Determine the equation of the least-squares regression line. Give interpretation of parameters' a and b values.
5. Make a prediction of Y-value if $X=2000$.
Make a conclusion.

UNIT 8



Time series analysis

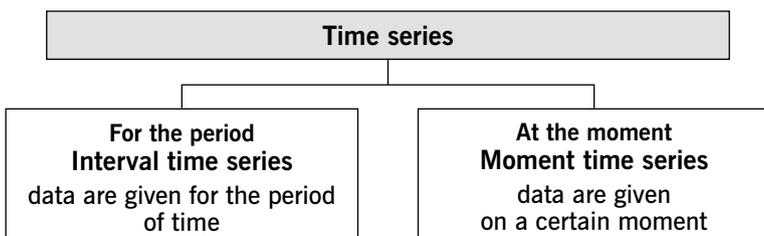
Learning objectives:

- Types of time series
- Average series level
- Time series indicators and relationship between them
- Average time series indicators
- Components of time series
- Methods for measuring trend
- Smoothing methods
- Linear trend
- Seasonal component
- Measuring error

THEORY

Time series data – numerical data obtained at regular time intervals. Time intervals can be annually, quarterly, monthly, weekly, daily, hourly.

Time series plot is a two-dimensional plot of time series data.



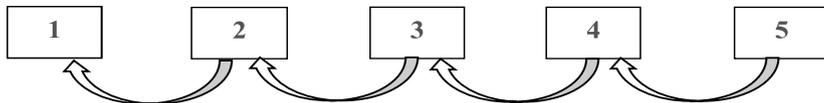
Average series level

Time series	Periods or moments	Formula	
Interval	equal	$\bar{y} = \frac{\sum y_i}{n}$	simple arithmetic mean
	unequal	$\bar{y} = \frac{\sum y_i \cdot t_i}{\sum t_i}$	weighted arithmetic man
Moment	equal	$\bar{y} = \frac{\frac{1}{2} \cdot y_1 + y_2 + \dots + \frac{1}{2} \cdot y_n}{n - 1}$	simple chronological mean
	unequal	$\bar{y} = \frac{(y_1 + y_2) \cdot t_1 + (y_2 + y_3) \cdot t_2 + \dots + (y_{n-1} + y_n) \cdot t_{n-1}}{2 \sum t_i}$	weighted chronological mean

Methods of calculating time series indicators

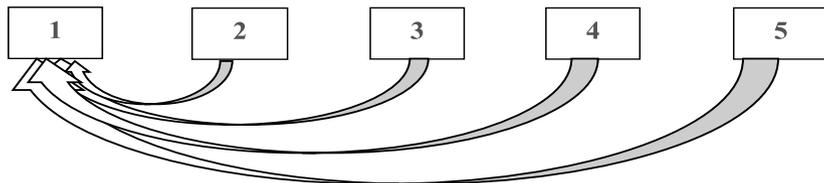
Comparison with variable base

Each current level is compared to previous level



Comparison with constant base

Each current level is compared to basic level



Time series indicators

Indicator	Russian equivalent	Formula	
		with variable base (Chain)	with constant base (Base)
Absolute growth, Δ	Абсолютный прирост	$\Delta = y_i - y_{i-1}$	$\Delta = y_i - y_0$

Indicator	Russian equivalent	Formula	
		with variable base (Chain)	with constant base (Base)
Growth coefficient, K_p	Кэффициент роста	$K_p = \frac{y_i}{y_{i-1}}$	$K_p = \frac{y_i}{y_0}$
Growth rate, T_p	Темп роста	$T_p = K_p \cdot 100\%$	$T_p = K_p \cdot 100\%$
Rate of increase (decrease), T_n	Темп прироста	$T_n = T_p - 100\%$ $T_n = (K_p - 1) \cdot 100\%$	$T_n = T_p - 100\%$ $T_n = (K_p - 1) \cdot 100\%$
Absolute value of 1% increase (decrease), A	Абсолютное значение 1% прироста	$A = \frac{\Delta}{T_n}$	$A = \frac{\Delta}{T_n}$

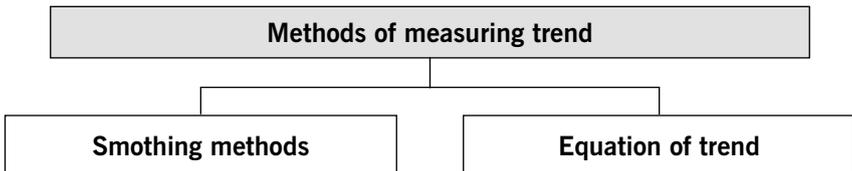
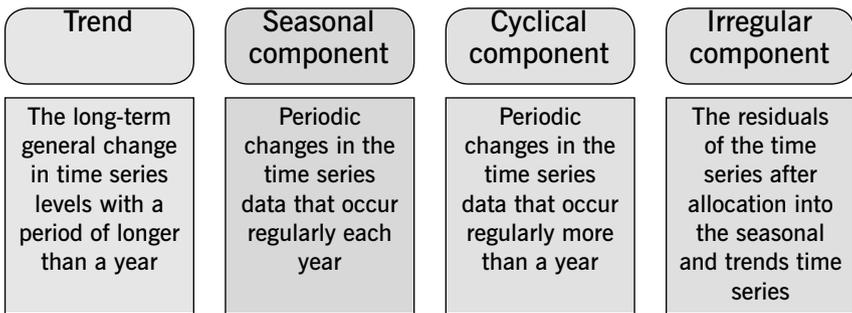
Relationship between time series indicators

- 1** $\Delta_{4/0} = \Delta_{1/0} + \Delta_{2/1} + \Delta_{3/2} + \Delta_{4/3}$
- 2** $K_{p4/0} = K_{p1/0} \cdot K_{p2/1} \cdot K_{p3/2} \cdot K_{p4/3}$
- 3** $K_{p4/3} = \frac{K_{p4/0}}{K_{p3/0}}$

Average Time Series Indicators

Indicator	Russian equivalent	Formula
Average absolute growth, $\bar{\Delta}$	Средний абсолютный прирост	$\bar{\Delta} = \frac{y_n - y_1}{n - 1}$ or $\bar{\Delta} = \frac{\Sigma \Delta}{n - 1}$
Average growth coefficient, \bar{K}_p	Средний коэффициент роста	$\bar{K}_p = \sqrt[n-1]{\frac{y_n}{y_1}}$ or $K_p = \sqrt[m]{K_{p1} \cdot K_{p2} \cdot \dots \cdot K_{pm}}$
Average growth rate, \bar{T}_p	Средний темп роста	$\bar{T}_p = \bar{K}_p \cdot 100\%$
Average rate of increase (decrease), \bar{T}_n	Средний темп прироста	$\bar{T}_n = \bar{T}_p - 100\%$ or $\bar{T}_n = (\bar{K}_p - 1) \cdot 100\%$
Average absolute value of 1% increase (decrease), \bar{A}	Абсолютное значение 1% прироста	$\bar{A} = \frac{\bar{\Delta}}{\bar{T}_n}$

Components of time series



Smoothing methods

1. Moving averages

- used for smoothing
- a series of arithmetic means over time
- result dependent upon length of period for calculating means (L)

$$\bar{y}_1 = \frac{y_1 + y_2 + \dots + y_n}{n}$$

$$\bar{y}_2 = \frac{y_2 + y_3 + \dots + y_{n+1}}{n}$$

and so on.

2. Exponential smoothing

- used for smoothing and short-term forecasting (one period in the future)

Methods for trend forecasting

Linear trend

Estimate a trend line using regression analysis:

$$\hat{y}_t = a_0 + a_1 \cdot t$$

y – the dependent variable for which trend values are to be calculated

t – time variable

a_0 and a_1 – parameters of the equation

Least squares method is used to define the values of parameters and

$$\begin{cases} na_0 + a_1 \sum t = \sum y \\ a_0 \sum t + a_1 \sum t^2 = \sum ty \end{cases}$$

Parameter t for time series with odd number of levels

Time series levels	y_1	y_2	y_3	y_4	y_5
Parameter t	-2	-1	0	+1	+2

Parameter t for time series with even number of levels

Time series levels	y_1	y_2	y_3	y_4	y_5	y_6
Parameter t	-5	-3	-1	+1	+3	+5

Formulas for parameters of linear equation

$$a_0 = \frac{\sum y}{n} \qquad a_1 = \frac{\sum yt}{\sum t^2}$$

Nonlinear trend

Quadratic form

$$\hat{y}_t = a_0 + a_1 t + a_2 t^2$$

Exponential trend model

$$\hat{y}_t = a_0 + a_1^t$$

Seasonal component

Seasonal component – periodic changes in the time series data that occur regularly each year

Seasonal indices:

1. Find averages for each month (January, February etc.) or for each quarter (\bar{y}_i)
2. Find average for all months or quarters (\bar{y}_0)

$$\bar{y}_0 = \frac{\sum y_i}{\text{number of months (quarters) in the period}}$$

3. Find seasonal indices (i_s)

$$i_s = \frac{\bar{y}_i}{\bar{y}_0} \cdot 100\%$$

Cyclical component

- Long-term wave-like patterns
- Regular occur but may vary in length
- Measured peak to peak or trough to trough

Irregular component

Irregular component – unpredictable, random, residual fluctuations.

Measuring error

Choose the model that gives the smallest measuring error

MAPE – Mean Absolute Percentage Error (Средняя ошибка аппроксимации)

$$MAPE = \frac{1}{n} \sum_{t=1}^n \left| \frac{y_t - \hat{y}_t}{y_t} \right| \cdot 100\%$$

The accuracy of the equation

MAPE	Accuracy
under 10%	high
10–20%	good
20–50%	satisfactory
50% and over	not satisfactory

TESTS

Test 8-1 (select a correct answer)

Average series level for moment time series with equal periods we count using the formula:

1. Simple arithmetic mean
2. Weighted arithmetic mean
3. Simple chronological mean
4. Weighted chronological mean

Test 8-2 (Match the component of time series in the left-hand column with its characteristic in the right-hand column)

1	Trend	a	The residuals of the time series after allocation into the seasonal and trends time series
2	Cyclical component	b	Periodic changes in the time series data that occur regularly each year
3	Irregular component	c	Periodic changes in the time series data that occur regularly more than a year
4	Seasonal component	d	The long-term general change in time series levels with a period of longer than a year

Test 8-3 (select a correct answer)

Given time series data:

Month	March	April	May	June	July
Oil production, thousand tons	920	880	878	870	868

For determining the average oil production for the whole period, we use the formula:

1. Simple arithmetic mean
2. Weighted arithmetic mean
3. Simple chronological mean
4. Weighted chronological mean

Test 8-4 (Match the parameter in the left-hand column with its formula in the right-hand column)

1	Average absolute growth	a	
2	Chain absolute growth	b	
3	Growth rate	c	
4	Base absolute growth	d	
5	Base growth coefficient	e	
6	Average growth coefficient	f	
7	Average rate of increase	g	
8	Chain growth coefficient	h	

Test 8-4 (select a correct answer)

The growth rate of the wage in the first year equal to 107,5% and in the second year – 108%. Determine total percentage increase of wages for two years?

- | | |
|-----------|-----------|
| 1. 115,5% | 3. 16,1% |
| 2. 15,5% | 4. 116,1% |

Test 8-5 (select a correct answer)

The growth rate of the wage in the first year equal to 107,5% and in the second year – 108%. Determine average percentage increase of wages for two years?

- | | |
|----------|----------|
| 1. 5,6% | 3. 16,1% |
| 2. 15,5% | 4. 7,7% |

Test 8-6 (select a correct answer)

Growth rate of sales is equal to 210%. It means that:

1. Sales increased by 11 times
2. Sales increased by 210%
3. Sales increased by 110%
4. Sales increased by 21 times

Test 8-7 (select a correct answer)

Given the deposit balances at the beginning of each month, thousand rubles:

01.01. – 200

01.02. – 220

01.03 – 180

01.04 – 260

Count the average deposit balance for the first quarter.

- | | |
|--------|--------|
| 1. 210 | 3. 215 |
| 2. 250 | 4. 200 |

Test 8-8 (select a correct answer)

The average actual export price of oil in December amounted to 89,8% by November. This value is:

1. Rate of increase
2. Growth coefficient
3. Absolute growth
4. Growth rate

Test 8-9 (select a correct answer)

Given time series data on aluminum production for 5 years:

Year	1	2	3	4	5
Production, thousand tons	46	31	20	18	17

Define the percentage of the decrease of production in the fifth year compared to the first year.

- | | |
|--------|--------|
| 1. 63% | 3. 37% |
| 2. 29% | 4. 36% |

Test 8-10 (select a correct answer)

Given time series data:

	01.01	01.02	01.03	01.04	01.05
Oil production, thousand tons	920	880	878	870	868

For determining the average oil production for the whole period, we use the formula:

1. Simple arithmetic mean
2. Weighted arithmetic mean
3. Simple chronological mean
4. Weighted chronological mean

PRACTICE

8.1. Given the data on hotels and restaurants in Russia (for the end of the year):

End of the year	2015	2016	2017	2018
Number of hotels and restaurants, thousand	59	64	68	80

Perform the following tasks:

1. Define the type of time series.
2. Draw time series plot.
3. Determine the average series level.
4. Calculate:
 - 1) time series indicators with variable base;
 - 2) time series indicators with constant base;
 - 3) average time series indicators.

8.2. There are data on the population, thou. persons:

On the 1st of January	2013	2014	2015	2016	2017
Moscow	11980	12108	12197	12330	12381
Saint-Petersburg	5028	5132	5192	5226	5282

Source: Regions of Russia. Socio-economic indicators. 2017

Perform the following tasks:

1. Define the average population size of Moscow and Saint-Petersburg for the period 2013-2017.
2. Calculate the average absolute value of 1% increase of population.

8.3. Given the data on oil production:

Year	2013	2014	2015	2016	2017
Oil production, thousand tons	420	440	452	458	471

Perform the following tasks:

1. Define the type of time series.
2. Determine the average series level.
3. Calculate average time series indicators.

8.4. Rosstat data on the number of unemployed is the following:

**Number of unemployed
in Russia in 2018**

	at the beginning of the month						
	January	February	March	April	May	June	July
Number of unemployed, thou. persons	4288	4226	4109	4050	3945	3846	3905

Source: Russian Federation State Statistical Service

Perform the following tasks:

1. Define the type of time series.
2. Calculate the average number of unemployed:
 - 1) in January;
 - 2) in the I quarter;
 - 3) in the II quarter;
 - 4) in the first half.

8.5. Rosstat data on the number of pensioners registered at the system of the Pension Fund of the Russian Federation is the following:

	at the beginning of the year				
	2013	2014	2015	2016	2017
Number of pensioners, thousand persons	38 411	39 706	41 456	42 729	43 177

Source: The Russian Statistical Yearbook, 2017, t.1.1 Russian Federation State Statistical Service

Perform the following tasks:

1. Define the type of time series.
2. Calculate for the period 2013-2017:
 - a) the average number of pensioners;
 - b) the average absolute growth;
 - c) the average growth coefficient;
 - d) average rate of increase.

8.6. Rosstat data on emissions of one of the most widespread air pollutants from stationary sources is the following:

	2000	2010	2014	2015	2016
Hard substances, thou. tones	2972,2	2381,2	1922,2	1820,4	1723,9

Source: The Russian Statistical Yearbook, 2017, t.3.14 Russian Federation State Statistical Service

Perform the following tasks:

1. Define the type of time series.
2. Calculate for the period 2014-2016:
 - a) absolute growth (decrease) with variable base (chain) and with constant base (base);
 - b) growth coefficient, growth rate and rate of increase; (decrease) with variable base (chain) and with constant base (base);
 - c) the average absolute growth (decrease);
 - d) the average growth coefficient and the average rate of increase (decrease).
3. Calculate for periods 2000-2010 and 2010-2014:
 - a) the average absolute growth (decrease);
 - b) the average growth coefficient and the average rate of increase (decrease).
4. Compare the results.

8.7. There are the following data on the balance of goods in the store:

- 01.01.18 – the balance was 400 tons
- 16.01.18 – received 450 tons
- 28.01.18 – shipped 300 tons
- 03.02.18 – received 220 tons
- 27.02.18 – received 100 tons
- 03.03.18 – shipped 330 tons
- 23.03.18 – received 700 tons

Define the average balance of goods in the store:

- a) for January;
- b) for February;
- c) for March, d) for first quarter.

8.8. Rosstat data on emissions of air pollutants from stationary sources is the following (by economic activity):

thou. tones

	2010	2011	2012	2013	2014	2015
Pollutants emitted into atmosphere, total	19116	19162	19630	18447	17452	17296
of which by economic activity:						
Agriculture, hunting and forestry	137	141	163	165	185	197
Mining and quarrying	5200	5616	6128	5266	4944	4755
Manufacturing	6431	6523	6407	6219	5932	5969
Electricity, gas and water supply	4327	4071	4164	3869	3762	3672
Transport and communications	2426	2248	2107	2220	1931	1885
Other community, social and personal service activities	108	166	232	263	286	326

Source: The Russian Statistical Yearbook, 2016, t..3.15 Russian Federation State Statistical Service

Perform the following tasks as a whole and for each type of economic activity:

1. Calculate absolute growth with variable base (chain) and with constant base (base).
2. Calculate growth coefficient, growth rate and rate of increase. with variable base (chain) and with constant base (base).
3. Calculate the average absolute growth.
4. Calculate the average growth coefficient and the average rate of increase.
5. Determine the parameters of the trend equation.
6. Determine mean absolute percentage error (MAPE) and make a conclusion about the accuracy of equation.
7. Make a forecast of the volume of emissions of air pollutants for 2018 using the trend equation.

8.9. Given the data on the freight turnover of transport:

	2014	2015	2016	2017
Absolute growth, mln. tone-km to the previous year	+2,8	+3,6	+4,3	+5,2

It is known that in 2015 the turnover increased by 4,5% compared to 2014.

Calculate for the period 2013-2017:

- the average turnover;
- the average absolute growth;
- the average growth coefficient;
- average rate of increase;
- the rate of increase in 2017 compared to 2013.

8.10. The freight turnover of transport increased by 2,8% annually on average from 2000 to 2010, and by 4,6% from 2010 to 2014. In 2015, the freight turnover remained at the level of 2014, and in 2016, compared to 2015, it increased by 5,6%.

Define the average growth coefficient for the periods:

- 2000-2014;
- 2000-2016.

8.11. It is known that 50 cars were sold in the salon in August 2018. In December 2018 – 70 cars.

Determine the possible number of car sales in February 2019, provided that the unchanged:

- the average monthly absolute growth;
- the average monthly growth rate.

8.12. Given the data on paid services rendered to population:

Year	Paid services rendered to population, thousand rubles	Compared to the previous year			
		Absolute growth, thousand rubles	Growth rate, %	Rate of increase, %	Absolute value of 1% increase, thousand rubles
2013	2454,0				
2014			104,5		
2015		+ 97,5			
2016					
2017		+ 80,2			26,619
2018				+ 3,6	

Perform the following tasks:

- Fill in the table with the missing indicators.
- Calculate for the period 2013-2018:
 - the average series level;
 - the average absolute growth
 - the average growth coefficient
 - average rate of increase

3. Make a forecast of the volume of paid services rendered to population, based on:

- a) the average absolute growth;
- b) the average growth rate;
- c) the trend equation.

8.13. For the Russian Federation and the countries of the EU countries there are data on changes in food prices (in % to the previous month):

	October 2018	November 2018	December 2018
Russia	101,3	102,6	103,9
EU countries	99,8	99,9	98,7

Define for Russia and EU countries:

- 1) change in food prices from September to December 2018;
- 2) average change in food prices.

8.14. Given the data on the price of sugar (per kilogram):

	2016	2017	2018
Rate of increase, % to the previous year	+9,5	+12,3	+20,0

Define the price of sugar in 2019 if it is known that it was 36,50 rubles in 2015.

8.15. Given the rate of increase of income:

- in the second quarter compared to the first – +2,8%
- in the third quarter compared to the second – +0,5%
- in the fourth quarter compared to the third – +1,3%.

Calculate the change of income:

- in the fourth quarter compared to the first;
- in the third quarter compared to the first.

8.16. The analysis of sales of the company selling soft drinks and seasonality influence on it is carried out. Given the following data:

Quarter	Sales, thousand liters		
	2016	2017	2018
I quarter	240	250	260
II quarter	320	330	340
III quarter	450	440	460
IV quarter	290	300	310
Total for a year:	1300	1320	1370

Perform the following tasks:

1. Define seasonal indices for each quarter.
2. Calculate the quarterly sales of soft drinks in 2019, if it is planned that the annual amount will be 1440 liters.

8.17. There are the following data on revenues from sales of travel agencies' services (million rubles):

Month	Years		
	2016	2017	2018
January	15	16	15
February	13	12	12
March	10	11	10
April	11	12	13
May	14	15	14
June	18	20	17
July	18	21	19
August	19	22	22
September	18	20	20
October	17	16	17
November	16	16	15
December	19	17	19

Perform the following tasks:

1. Define seasonal indices for each month.
2. Determine the monthly revenue from sales in 2019, if it is known that the total amount in 2019 may be 240 million rubles.

8.18. Given the data on the volume of sales:

	Years					
	2012	2013	2014	2015	2016	2017
Volume of sales, thousand tons	47	50	52	55	58	60

Perform the following tasks: make a forecast of the volume of sales for 2018:

- 1) using the average absolute growth;
- 2) using the average growth coefficient;
- 3) using the trend line equation.

8.19. Given the data on the volume of sales:

	Years					
	2013	2014	2015	2016	2017	2018
Volume of sales, thousand tons	15	13	12	10	9	9

Perform the following tasks:

1. Make a forecast of the volume of sales for 2019 using the trend line equation.
2. Determine mean absolute percentage error (MAPE) and make a conclusion about the accuracy of equation.

8.20. The analysis of sales of the company selling soft drinks and seasonality influence on it is carried out. Given the following data:

Quarter	Sales, thousand liters		
	2016	2017	2018
I quarter	240	250	260
II quarter	320	330	340
III quarter	450	440	460
IV quarter	290	300	310
Total for a year:	1300	1320	1370

Perform the following tasks:

1. Define seasonal indices for each quarter.
2. Calculate the quarterly sales of soft drinks in 2019, if it is planned that the annual amount will be 1440 liters.

8.21. According to the following data, calculate the seasonality index for January, February, May:

	Revenue, thousand rubles	
	2017	2018
Total for a year:	4200	4350
including:		
January	290	300
February	330	340
May	370	368

8.22. Given the share prices of JSC Gazprom:

Month	Share price, rubles per share
12.2017	133
01.2018	143
02.2018	141
03.2018	140
04.2018	142
05.2018	146
06.2018	141
07.2018	142
08.2018	143
09.2018	154
10.2018	162

Source – MICEX

Perform the following tasks:

1. Calculate 3-month moving averages ($L=3$).
2. Draw actual and smoothed prices on time series plot.

8.23. Given the share prices of JSC Lukoil:

Month	Share price, US dollars per share
01-2017	55,6
02-2017	55,5
03-2017	52,9
04-2017	51,8
05-2017	49,5
06-2017	47,7
07-2017	48,3
08-2017	48,7
09-2017	51,6
10-2017	52,5
11-2017	55,9
12-2017	57,5
01-2018	65,2

Окончание

Month	Share price, US dollars per share
02-2018	65,8
03-2018	66,8
04-2018	65,6
05-2018	68,4
06-2018	66,6
07-2018	69,2
08-2018	68,1
09-2018	69,8
10-2018	73,8
11-2018	75,0

Source – MICEX

Perform the following tasks:

1. Calculate 5-month moving averages ($L=5$).
2. Draw actual and smoothed prices on time series plot.

8.24. World production of copper for 9 years is the following:

Year	Production of copper, million tons
2010	18,0
2011	18,5
2012	18,6
2013	19,2
2014	19,7
2015	20,2
2016	20,8
2017	22,9
2018	23,1

Source – WBMS World Bureau of Metal Statistics

Perform the following tasks:

1. Make a forecast of copper production for 2019 using the trend line equation.
2. Determine mean absolute percentage error (MAPE) and make a conclusion about the accuracy of equation.

8.25. Given the data on unemployment rates of leading countries:

Month -year	Unemployment rate, %		
	German	United Kingdom	USA
01-2018	5,4	4,3	4,1
02-2018	5,4	4,2	4,1
03-2018	5,3	4,2	4,1
04-2018	5,3	4,2	4,1
05-2018	5,2	4,2	3,9
06-2018	5,2	4,0	3,8
07-2018	5,2	4,0	4,0
08-2018	5,2	4,0	3,9
09-2018	5,1	4,1	3,7

Source: Federal Statistical Office (Destatis)

Source: Office for National Statistics

Source: Federal Reserve, Department of Commerce, Department of Labor

Perform the following tasks:

1. Determine average growth rate and compare the dynamics of the unemployment rates of leading countries.
2. Make a forecast of unemployment rates for 10-2018 using the trend line equation.
3. Determine mean absolute percentage error (MAPE) and make a conclusion about the accuracy of equation.



Index numbers

Learning objectives:

- Simple and aggregate indices
- Forms of price index
- Factor analysis
- Weighted average quantity and price indices
- Base and chain indices
- Average price index

THEORY

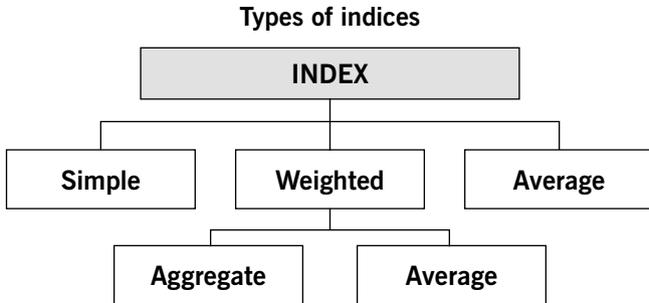
Index is a statistical measure of changes in variable or a group of related variables over time and territories.

Index method allows to:

1. Measure the changes in variable over time
2. Measure the changes in variable over geographical location
3. Analyze the influence of the factors on the change of the variable

Notations

Parameter	Russian equivalent	Variable
p	Цена за единицу продукции	Price
q	Количество продукции	Quantity
pq	Стоимость продукции (выручка от реализации, товарооборот)	Value (sales proceeds, trade turnover)
z	Себестоимость единицы продукции	Cost per unit
zq	Затраты на выпуск продукции	Total cost



Simple Indices
(computed for each commodity)

Aggregate Indices
(computed for all commodities)

Simple index	Russian equivalent	Formula	Aggregate index	Russian equivalent	Formula
Quantity index	Индекс физическо-го объема	$i_{q_{1/0}} = \frac{q_1}{q_0}$	Quantity index	Индекс физическо-го объема	$i_{q_{1/0}} = \frac{\sum q_1 p_0}{\sum q_0 p_0}$
Price index	Индекс цен	$i_{p_{1/0}} = \frac{p_1}{p_0}$	Price index	Индекс цен	$i_{p_{1/0}} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$
Unit cost index	Индекс себестоимости	$i_{z_{1/0}} = \frac{z_1}{z_0}$	Unit cost index	Индекс себестоимости	$i_{z_{1/0}} = \frac{\sum z_1 q_1}{\sum z_0 q_1}$
Value index	Индекс стоимости	$i_{pq_{1/0}} = \frac{p_1 q_1}{p_0 q_0}$	Value index	Индекс стоимости	$i_{pq_{1/0}} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$
Total cost index	Индекс затрат	$i_{zq_{1/0}} = \frac{z_1 q_1}{z_0 q_0}$	Total cost index	Индекс затрат	$i_{zq_{1/0}} = \frac{\sum z_1 q_1}{\sum z_0 q_0}$

Forms of price index

Price index	Russian equivalent	Formula
Paasche price index	Индекс цен Пааше	$i_{p_{1/0}} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$
Laspeyres price index	Индекс цен Ласпейреса	$i_{p_{1/0}} = \frac{\sum p_1 q_0}{\sum p_0 q_0}$
Fisher price index	Индекс цен Фишера	$i_{p_{1/0}} = \sqrt{\frac{\sum p_1 q_1}{\sum p_0 q_1} \cdot \frac{\sum p_1 q_0}{\sum p_0 q_0}}$

Factor Analysis

Relative change of the value (pq)

$$i_{pq_{1/0}} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$$

Absolute change of the value

$$\Delta_{pq} = \sum p_1 q_1 - \sum p_0 q_0$$

<p>Change of prices (p)</p> <ul style="list-style-type: none"> Average change of prices of all commodities $I_{p_{1/0}} = \frac{\sum p_1 q_1}{\sum p_0 q_1}$ <ul style="list-style-type: none"> Absolute change of value due to change of prices: $\Delta_{pq}^p = \sum p_1 q_1 - \sum p_0 q_1$	<p>Change of quality(q)</p> <ul style="list-style-type: none"> Change of quality of all commodities $I_{q_{1/0}} = \frac{\sum q_1 p_0}{\sum q_0 p_0}$ <ul style="list-style-type: none"> Absolute change of value due to change of quantity: $\Delta_{pq}^q = \sum q_1 p_0 - \sum q_0 p_0$
$\Delta_{pq} = \Delta_{pq}^p + \Delta_{pq}^q$	

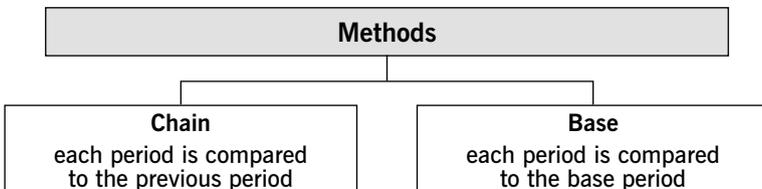
Weighted average quantity indices

Arithmetic	Harmonic
Simple quantity indices and value in base period are given	Simple quantity indices and value in current period are given
$I_{q_{1/0}} = \frac{\sum i_{q_{1/0}} \cdot q_0 p_0}{\sum q_0 p_0}$	$I_{q_{1/0}} = \frac{\sum q_1 p_1}{\sum \frac{q_1 p_1}{i_{q_{1/0}}}}$

Weighted average price indices

Arithmetic	Harmonic
Simple price indices and value in base period are given	Simple price indices and value in current period are given
$I_{p_{1/0}} = \frac{\sum i_{p_{1/0}} \cdot p_0 q_0}{\sum p_0 q_0}$	$I_{p_{1/0}} = \frac{\sum p_1 q_1}{\sum \frac{p_1 q_1}{i_{p_{1/0}}}}$

Methods of calculating indices



System of base simple indices

Simple Index	Base indices
Quantity index	$i_{q_{1/0}} = \frac{q_1}{q_0}; i_{q_{2/0}} = \frac{q_2}{q_0}; i_{q_{3/0}} = \frac{q_3}{q_0}$
Price index	$i_{p_{1/0}} = \frac{p_1}{p_0}; i_{p_{2/0}} = \frac{p_2}{p_0}; i_{p_{3/0}} = \frac{p_3}{p_0}$
Value index	$i_{pq_{1/0}} = \frac{p_1q_1}{p_0q_0}; i_{pq_{2/0}} = \frac{p_2q_2}{p_0q_0}; i_{pq_{3/0}} = \frac{p_3q_3}{p_0q_0}$

System of chain simple indices

Simple Index	Base indices
Quantity index	$i_{q_{1/0}} = \frac{q_1}{q_0}; i_{q_{2/1}} = \frac{q_2}{q_1}; i_{q_{3/2}} = \frac{q_3}{q_2}$
Price index	$i_{p_{1/0}} = \frac{p_1}{p_0}; i_{p_{2/1}} = \frac{p_2}{p_1}; i_{p_{3/2}} = \frac{p_3}{p_2}$
Value index	$i_{pq_{1/0}} = \frac{p_1q_1}{p_0q_0}; i_{pq_{2/1}} = \frac{p_2q_2}{p_1q_1}; i_{pq_{3/2}} = \frac{p_3q_3}{p_2q_2}$

Instead the price (p) may be the unit cost (z)

Relationship between Base and Chain Indices

1. Base simple index is equal to multiplication of chain simple indices:

$$i_{3/0} = i_{1/0} \cdot i_{2/1} \cdot i_{3/2}$$

2. Chain simple index is equal to the division of two base simple indices:

$$i_{3/2} = \frac{i_{3/0}}{i_{2/0}}$$

System of base aggregate indices

Simple Index	Base indices
Quantity index	$i_{q_{1/0}} = \frac{\sum q_1 p_0}{\sum q_0 p_0}; i_{q_{2/0}} = \frac{\sum q_2 p_0}{\sum q_0 p_0}; i_{q_{3/0}} = \frac{\sum q_3 p_0}{\sum q_0 p_0}$
Price index	$i_{p_{1/0}} = \frac{\sum p_1 q_1}{\sum p_0 q_1}; i_{p_{2/0}} = \frac{\sum p_2 q_2}{\sum p_0 q_2}; i_{p_{3/0}} = \frac{\sum p_3 q_3}{\sum p_0 q_3}$
Value index	$i_{pq_{1/0}} = \frac{\sum p_1 q_1}{\sum p_0 q_0}; i_{pq_{2/0}} = \frac{\sum p_2 q_2}{\sum p_0 q_0}; i_{pq_{3/0}} = \frac{\sum p_3 q_3}{\sum p_0 q_0}$

System of chain aggregate indices

Simple Index	Base indices
Quantity index	$i_{q_{1/0}} = \frac{\sum q_1 p_0}{\sum q_0 p_0}; i_{q_{2/1}} = \frac{\sum q_2 p_1}{\sum q_1 p_1}; i_{q_{3/2}} = \frac{\sum q_3 p_2}{\sum q_2 p_2}$
Price index	$i_{p_{1/0}} = \frac{\sum p_1 q_1}{\sum p_0 q_1}; i_{p_{2/1}} = \frac{\sum p_2 q_2}{\sum p_1 q_2}; i_{p_{3/2}} = \frac{\sum p_3 q_3}{\sum p_2 q_3}$
Value index	$i_{pq_{1/0}} = \frac{\sum p_1 q_1}{\sum p_0 q_0}; i_{pq_{2/1}} = \frac{\sum p_2 q_2}{\sum p_1 q_1}; i_{pq_{3/2}} = \frac{\sum p_3 q_3}{\sum p_2 q_2}$

Indices of average qualitative indicators (price, unit coast)

Index of average price \bar{p} (Index of variable structure)

$$I_{\bar{p}_{1/0}} = \frac{\bar{p}_1}{\bar{p}_0} = \frac{\sum p_1 q_1}{\sum q_1} : \frac{\sum p_0 q_0}{\sum q_0} = \frac{\sum p_1 d_1}{\sum p_0 d_0}$$

Absolute change of average price \bar{p} :

$$\Delta_{\bar{p}} = \bar{p}_1 - \bar{p}_0 = \frac{\sum p_1 q_1}{\sum q_1} - \frac{\sum p_0 q_0}{\sum q_0} = \sum p_1 d_1 - \sum p_0 d_0$$

Index of change of average price \bar{p} due to chance of prices p (Index of constant structure)

$$I_{\bar{p}_{1/0}}^p = \frac{\sum p_1 q_1}{\sum q_1} : \frac{\sum p_0 q_1}{\sum q_1} = \frac{\sum p_1 d_1}{\sum p_0 d_1}$$

Absolute change of average price \bar{p} due to chance of prices p :

$$\Delta_{\bar{p}}^p = \frac{\sum p_1 q_1}{\sum q_1} - \frac{\sum p_0 q_1}{\sum q_1} = \sum p_1 d_1 - \sum p_0 d_1$$

Index of change of average price \bar{p} due to chance of the structure of q (Index of structural shifts)

$$I_{\bar{p}_{1/0}}^q = \frac{\sum p_0 q_1}{\sum q_1} : \frac{\sum p_0 q_0}{\sum q_0} = \frac{\sum p_0 d_1}{\sum p_0 d_0}$$

Absolute change of average price \bar{p} due to chance of the structure of q :

$$\Delta_{\bar{p}}^q = \frac{\sum p_0 q_1}{\sum q_1} - \frac{\sum p_0 q_0}{\sum q_0} = \sum p_0 d_1 - \sum p_0 d_0$$

Relationship between Index of variable structure,
Index of constant structure and Index of structural shifts

$$I_{\bar{p}_{1/0}} = I_{\bar{p}_{1/0}}^p \cdot I_{\bar{p}_{1/0}}^q$$

$$\Delta_{\bar{p}} = \Delta_{\bar{p}}^p + \Delta_{\bar{p}}^q$$

TESTS

Test 9-1 (Match the index in the left-hand column with its formula in the right-hand column):

1	Simple quantity index	a	$i_{z_{1/0}} = \frac{z_1}{z_0}$
2	Aggregate value index	b	$i_{p_{1/0}} = \frac{p_1}{p_0}$
3	Total cost index	c	$I_{q_{1/0}} = \frac{\sum q_1 p_0}{\sum q_0 p_0}$
4	Simple unit cost index	d	$I_{p_{1/0}} = \frac{\sum p_1 q_1}{\sum p_0 q_1}$
5	Aggregate price index	e	$i_{q_{1/0}} = \frac{q_1}{q_0}$
6	Aggregate quantity index	f	$I_{zq_{1/0}} = \frac{\sum z_1 q_1}{\sum z_0 q_0}$
7	Simple cost index	g	$I_{pq_{1/0}} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$
8	Simple price index	h	$i_{zq_{1/0}} = \frac{z_1 q_1}{z_0 q_0}$

Test 9-2 (select a correct answer)

Choose the correct formula for weighted average arithmetic price index:

$$1. I_{q_{1/0}} = \frac{\sum i_{q_{1/0}} q_0 p_0}{\sum q_0 p_0}$$

$$2. I_{p_{1/0}} = \frac{\sum p_1 q_1}{\sum \frac{p_1 q_1}{i_{p_{1/0}}}}$$

$$3. I_{q_{1/0}} = \frac{\sum q_1 p_1}{\sum \frac{q_1 p_1}{i_{p_{1/0}}}}$$

$$4. I_{p_{1/0}} = \frac{\sum i_{p_{1/0}} p_0 q_0}{\sum p_0 q_0}$$

Test 9-3 (select a correct answer)

Choose the correct formula for weighted average harmonic quantity index:

$$1. I_{q1/0} = \frac{\sum i_{q1/0} q_0 p_0}{\sum q_0 p_0}$$

$$2. I_{p1/0} = \frac{\sum p_1 q_1}{\sum \frac{p_1 q_1}{i_{p1/0}}}$$

$$3. I_{q1/0} = \frac{\sum q_1 p_1}{\sum \frac{q_1 p_1}{i_{q1/0}}}$$

$$4. I_{p1/0} = \frac{\sum i_{p1/0} p_0 q_0}{\sum p_0 q_0}$$

Test 9-4 (select a correct answer)

Determine the missing indicators in the formula for Paasche price index:

$$I_{p1/0} = \frac{\sum p_1 q_{\quad}}{\sum p_0 q_{\quad}}$$

1. 1, 0

3. 1, 1

2. 0, 1

4. 0, 0

Test 9-5 (select a correct answer)

Name the formula:

$$I_{z1/0} = \frac{\sum z_1 q_1}{\sum z_0 q_1}$$

1. Aggregate quantity index

2. Simple unit cost index

3. Aggregate unit cost index

4. Aggregate total cost index

Test 9-6 (select a correct answer)

Simple price index = 120%, simple quantity index = 105%. Determine simple value index.

1. 125%

3. 126%

2. 225%

4. 115%

Test 9-7 (select a correct answer)

The quantity of the commodities increased in the second quarter compared to the first quarter by 5,0%, and total cost decreased by 10,0%.

Define the average change of unit cost in the second quarter compared to the first quarter.

1. 5,0%
2. 15,0%
3. 4,8%
4. 3,7%

Test 9-8 (select a correct answer)

Commodity	Price		Quantity	
	p_0	p_1	q_0	q_1
A	5	6	10	14
B	4	7	14	16

Determine simple price index of commodity A.

1. 120,0%
2. 175,0%
3. 140,0%
4. 110,0%

Test 9-9 (select a correct answer)

Commodity	Price		Quantity	
	p_0	p_1	q_0	q_1
A	5	6	10	12
B	4	7	14	16

Determine aggregate price index.

1. 152,6%
2. 148,4%
3. 143,7%
4. 120,5%

Test 9-10 (select a correct answer)

Absolute change of value due to change of prices = 100 thousand rubles, absolute change of value = 300 thousand rubles.

Determine absolute change of value due to quantity.

1. 200
2. 400
3. 100
4. 500

PRACTICE

9.1. There is data on exports of non-ferrous metals:

Non-ferrous metals	Export price per 1 ton, thousand US dollars		Quantity of export, thousand tons	
	2017	2018	2017	2018
Copper	6	5	200	280
Nickel	20	14	250	240
Aluminum	3	2	340	360

Determine:

1. Simple price, quantity and value indices.
2. Aggregate price, quantity and value indices.
3. Absolute change of value:
 - overall;
 - due to change of prices;
 - due to change of quantity.

9.2. There are data on the production of three commodities:

Commodity	Unit cost, rub		Quantity, units	
	2016	2018	2016	2018
A	250	220	180	190
B	310	320	170	160
C	360	340	130	140

Determine:

1. Simple unit cost, quantity and total cost indices.
2. Aggregate unit cost, quantity and total cost indices.
3. Absolute change of total cost:
 - overall;
 - due to change of unit cost;
 - due to change of quantity.

9.3. Given the following data:

Commodity	Quantity, thousand tons		Total cost, million rubles	
	Quarter I	Quarter II	Quarter I	Quarter II
A	406	390	130	110
	500	400	260	220

Define aggregate quantity index.

9.4. The reporting data on the refinery for 2018 are the following:

Commodity	Price per 1 ton, thou. tons		Quantity, thou. tons	
	February	March	February	March
Gasoline AI-95	26	28	45	50
Diesel fuel	20	21	125	130

Calculate the absolute change of value due to change of prices.

9.5. Given the following data:

Commodity	Trade turnover in September, thou. rub.	Simple price index in September compared to July, %
A	1650	103,7
B	2610	107,4
C	1195	106,3

Calculate:

1. Weighted average price index.
2. Aggregate quantity index if it is known that trade turnover increased by 12% in September compared to July.

9.6. There are the following data on export of oil and gas by the Russian Federation:

Commodity	Value of export products, mln. US dollars		Change of quantity in 2018 compared to 2017, %
	2017	2018	
Oil	102 100	212 200	+4,0
Gas	43 800	44 900	-5,0

Determine:

1. Aggregate value index.
2. Change in quantity of all commodities using an appropriate index.
3. Absolute change of value of export products due to change of quantity.

9.7. Given the data on producing steel structures:

Commodity	Total cost, mln. rub.		Change of quantity in April compared to March, times
	March	April	
Fundamental blocks	80	82	1,025
Floor slab	185	186	0,985

Calculate the absolute change of total cost due to changes in quantity.

9.8. Given the data on export of cars to foreign countries in 2016:

	Quantity, thou. units			Export sales proceed, mln. US dollars		
	January	February	March	January	February	March
Cars	7	11	14	63	110	140
Trucks	0,5	1,4	1,5	15	49	57

Define chain aggregate price, quantity and sales proceed indices.

9.9. Given the data on external trade turnover of Russian Federation:

	2014	2015	2016	2017	2018
External trade turnover, percent to the previous year:	103.5	100.0	93.2	66.4	88.6
– Exports	102.3	98.9	95.2	68.7	82.5
– Imports	105.4	101.6	90.2	62.7	99.2

Determine the change of the external trade turnover (including exports and imports) in 2018 year compared to 2013 year.

9.10. The quantity of product increased in the second quarter compared to the first quarter by 2,8%, in the third quarter compared to the second quarter increased by 1,4%, in the fourth quarter compared to the third quarter decreased by 2,2%.

Define the change of the product quantity in the fourth quarter compared to the first quarter.

9.11. Total cost in the first quarter – 8000 thou. rub, in the second quarter – 8700 thou. rub, in the third quarter – 9000 thou. rub.

Unit cost decreased in the second quarter compared to the first quarter on average by 3,5%, and in the third quarter compared to the second quarter increased by 1,8%.

Determine aggregate quantity index in the second quarter compared to the first and in the third quarter compared to the second.

9.12. There are data on the implementation of the bond in three branches of the Bank:

Branch	Bond price, thousand rubles		Value, million rubles	
	2017	2018	2017	2018
1	15,0	14,5	30,0	26,1
2	20,0	22,0	28,0	35,2
3	25,0	23,5	27,5	28,2

Determine:

1. The index of average bond price.
2. Absolute change of average price.

9.13. Data are available for two construction firms:

Firm	2017		2018	
	Unit cost of 1 sq. m., thou. rub	Quantity, thousand sq. m	Unit cost of 1 sq. m., thou. rub	Quantity, thousand sq. m
1	435	186	460	168
2	612	92	652	135

Determine the absolute change of average unit cost.

9.14. There are data on export of three commodities:

Commodity	Export value, thousand rubles		Quantity of export, kg	
	2018	2019	2018	2019
A	120	154	200	280
B	225	240	250	240
C	102	90	340	360

Determine:

1. Paasches aggregate price index.
2. Absolute change of value due to change of prices.

9.15. There are data on the production of three commodities:

Commodity	Total cost, thou. rub		Quantity, units	
	October	December	October	December
A	45,0	41,8	180	190
B	52,7	51,2	170	160
C	46,8	47,6	130	140

Determine:

1. Aggregate quantity index.
2. Absolute change of total cost due to change of unit cost and quantity.

9.16. There are the following data about the sale of clothing:

Clothing	Trade turnover, thousand rubles			Change of quantity in April compared to March, %	Change of quantity in May compared to April, %
	March	April	May		
Everyday	1120	1125	1260	+5,0	+4,2
Exclusive	850	865	870	-2,5	-1,3
Sport	942	967	975	+1,5	+3,6

Calculate:

1. Base aggregate turnover indices (%)
2. Change of quantity of every type of clothing in May compared to March (%)
3. Weighted average quantity indices in April compared to March and in May compared to April.

9.17. Given the data on trade turnover:

	2015	2016	2017	2018	2019
Trade turnover, % to 2010 year:	90,5	98,3	101,4	103,6	105,6

Determine the change of trade turnover in 2019 year compared to 2017 year and in 2018 year compared to 2014 year.

9.18. The quantity of the commodities increased in the second quarter compared to the first quarter by 4,6%, and total cost decreased by 1,5%.

Define the average change of unit cost in the second quarter compared to the first quarter.

9.19. There are data on sales of butter in three supermarkets in 2018:

Supermarket	Price, rubles per kg		Quantity, kg	
	January	March	January	March
1	513	520	100	107
2	526	530	98	102
3	499	515	120	125

Determine the absolute change of average price in March compared to January as a whole and due to each factor:

- a) changes in the structure of sales;
- b) changes in prices in supermarkets.

9.20. Data on the production of office chairs by three enterprises are the following:

Enterprise	2017		2018	
	Total cost, thou. rub	Unit cost of 1 chair, thou. rub	Total cost, thou. rub	Unit cost of 1 chair, thou. rub
1	1085	3,5	1024	3,2
2	1890	4,2	2093	4,6
3	1450	5,0	1470	4,9

Determine the index of average unit cost.

9.21. Data on average monthly nominal accrued wages of employees of the two branches of the organization are as follows:

Branches	The average monthly nominal accrued wages, thou. rub		Number of employees, persons	
	2017	2018	2017	2018
1	1085	3,5	1024	3,2
2	1890	4,2	2093	4,6

Determine for the organization as a whole:

- 1) the relative change in average wages;
- 2) the absolute change in average wages;
- 3) the impact of each factor on the change in average wages:
 - a) structural changes in the number of personnel of the organization;
 - b) changes in the level of wages.

9.22. There are the following data on two commodities:

Commodity	Value, million rubles	
	February	April
A	1920	1940
B	1118	1290

Price of commodity A decreased by 7,6% and price of commodity B increased by 1,4% in April compared to February.

Determine the average change of prices and change of quality of commodities.

9.23. There are the following data on three commodities:

Commodity	Price per unit, rubles		Quantity, thousand units		Trade turnover, thousand rubles	
	2017	2018	2017	2018	2017	2018
A	100	110	520	540	52000	59400
B	120	125	610	620	73200	77500
C	140	155	400	410	56000	63550

Determine:

1. Simple price, quantity and value indices.
2. Aggregate price, quantity and value indices.
3. Absolute change of value:
 - overall;
 - due to change of prices;
 - due to change of quantity.

Additional costs of buyers due to rising prices

9.24. There are the following data on three commodities:

Commodity	Trade turnover in 2017, thousand rubles	Change of quantity in 2018 compared to 2017, %	Change of prices in 2018 compared to 2017, %
A	1200	+12,5	-4,0
B	1800	-3,8	+8,3
C	1000	+4,9	+2,2

Determine:

1. Simple price, quantity and value indices.
2. Average change of prices of all commodities.
3. Change of quality of all commodities.
4. Aggregate value index.
5. Absolute change of value:
 - overall;
 - due to change of prices;
 - due to change of quantity.

9.25. There are the following data on three commodities:

Commodity	Trade turnover in 2018, thousand rubles	Change of quantity in 2018 compared to 2017, %	Change of prices in 2018 compared to 2017, %
A	1100	+8,5	-3,3

Окончание

Commodity	Trade turnover in 2018, thousand rubles	Change of quantity in 2018 compared to 2017, %	Change of prices in 2018 compared to 2017, %
B	1500	-9,8	+6,4
C	900	+5,2	-1,8

Determine:

1. Simple price, quantity and value indices.
2. Aggregate price, quantity and value indices.
3. Absolute change of value:
 - overall;
 - due to change of prices;
 - due to change of quantity.

Additional costs of buyers due to rising prices.

9.26. There are the following data on three commodities:

Commodity	Price per unit, rubles		Quantity, thousand units	
	2017	2018	2017	2018
A	120	126	100	120
B	80	88	120	130
C	60	65	200	220

Determine:

- 1) the value of commodities at current prices in each year;
- 2) the value of commodities in 2018 at the prices of 2017 (at comparable prices);
- 3) the changes in the value of commodities in current prices in 2018 compared to 2017;
- 4) the changes in the value of commodities at comparable prices in 2018 compared to 2017;
- 5) average change of prices, quality and value of all commodities.

9.27. Trade turnover in 2018 amounted to 500 million rubles. Prices in 2018 compared to 2016 increased by 6.6 %.

Determine the trade turnover of 2018 at comparable prices (at 2016 prices).

9.28. Rosstat data on fixed capital investments for environmental protection and rational use of natural resources are the following:

	2014	2015	2016
Fixed capital investments- total			
• million rubles (at current prices)	158636,7	151788,0	139677,1
• percent of previous year (at comparable prices)	122,4	86,0	86,6

Source: The Russian Statistical Yearbook, 2017, t.3.15 Russian Federation State Statistical Service

Determine:

1. Aggregate value index of fixed capital investments (at current prices).
2. Aggregate value index of fixed capital investments (at comparable prices).
3. Aggregate price index.

UNIT 10



Population Statistics

Learning objectives:

- De-facto and de-jure population
- Age groups of population
- Dependency ratio
- Average annual population
- Natural increase of population
- Net migration
- Fertility rates
- Mortality and vital indicators
- Marriages and divorces
- Migration indicators

THEORY

Relationship between categories of population:

$$\text{De-facto population} = \text{De-jure population} + \text{Temporary residents} - \text{Temporary absent}$$

$$\text{De-jure population} = \text{De-facto population} - \text{Temporary residents} + \text{Temporary absent}$$

Main Age Groups

Age Groups		
Under working age children adolescents under 15 years	Working age males 16-64 years females 16-59 years	Over working age males 65 years and more females 60 years and more

Age Structure of Population

Age Structure – the number and proportion of every age group in total population

Age group	Population, persons	Age structure, %
Under working age		
Working age		
Over working age		
Total:		100,0

Dependency ratio (%o)

Child dependency ratio

$$K_{child} = \frac{\text{Under working aged population}}{\text{Working aged population}} \cdot 1000$$

Old-aged dependency ratio

$$K_{old-aged} = \frac{\text{Over working aged population}}{\text{Working aged population}} \cdot 1000$$

Total dependency ratio

$$K_{total} = \frac{\text{Under working aged population} + \text{Over working aged population}}{\text{Working aged population}} \cdot 1000$$

$$K_{total} = K_{child} + K_{old-aged}$$

Index of ageing

(percentage of population aged 60 and more in total population)

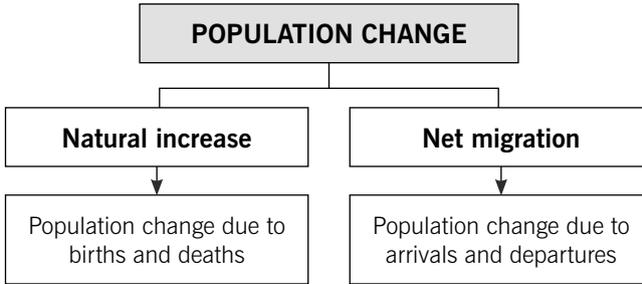
$$I_{ageing} = \frac{P_{60+}}{P} \cdot 100\%$$

Average annual population

(mid-point population)

$$\bar{P} = \frac{P_{beginning\ of\ the\ year} + P_{end\ of\ the\ year}}{2}$$

Components of population change



$$\Delta_{total} = \Delta_{natural} + \Delta_{net\ migration}$$

Fertility Rates

Indicator	Formula	Interpretation
Absolute number of birth	B	Number of births
Crude birth rate (Общий коэффициент рождаемости)	$Rate_{birth} = \frac{B}{P} \cdot 1000$	Number of births per 1000 population
Special birth Rate (Специальный коэффициент рождаемости)	$Rate_{birth\ special} = \frac{B}{\bar{P}_{women\ 15-49}} \cdot 1000$	Number of births per 1000 women aged 15-49
Age-specific fertility rate (Возрастной коэффициент рождаемости)	$Rate_{birth\ age_x} = \frac{B}{P_{women_x}} \cdot 1000$	Number of births per year to women of this age group to average annual number of women of this age
Total fertility rate (Суммарный коэффициент рождаемости)	$Rate_{total} = a \cdot \sum_{15}^{49} Rate_{birth\ age_x} \cdot 0,001$	The sum of age-specific fertility rates calculated for age groups in the interval 15-49 years. This rate shows how many births would give a single woman to children throughout the reproductive period

Mortality and Vital Indicators

Indicator	Formula	Interpretation
Absolute number of death	D	Number of deaths
Crude death rate (Общий коэффициент смертности)	$Rate_{death} = \frac{D}{P} \cdot 1000$	Number of deaths per 1000 population
Infant mortality rate (Коэффициент младенческой смертности)	$Rate_{infant} = \frac{D_{under\ the\ year}}{B_{for\ the\ year}} \cdot 1000$	Ratio of number of deaths of infants under one year to the number of birth
Natural increase (Естественный прирост)	$\Delta_{natural} = B - D$	Absolute value of difference between births and deaths. Its value may be either positive or negative
Natural increase rate (Коэффициент естественного прироста)	$Rate_{natural} = \frac{B - D}{P} \cdot 1000$ $Rate_{natural} = Rate_{birth} - Rate_{death}$	Characterizes the increase (decrease) of the population by 1000 people
Pokrovsky's vitality rate (Коэффициент жизнениности Покровского)	$Rate_{vitality} = \frac{B}{D}$	Characterizes the number of births by the number of deaths

Marriages and Divorces

Marriages	Marriages	Number of marriages
Marriage rate (Коэффициент брачности)	$Rate_{marriage} = \frac{Marriages}{P} \cdot 1000$	Number of marriages per 1000 population
Divorces	Divorces	Number of divorces
Divorce rate (Коэффициент разводимости)	$Rate_{divorce} = \frac{Divorces}{P} \cdot 1000$	Number of divorces per 1000 population
Stability of marriages (Устойчивость браков)	$Stability_{marriages} = \frac{Divorces}{Marriages} \cdot 100$	Number of divorces per 100 marriages

Migration indicators

Absolute indicators

Indicator	Formula	Interpretation
Arrivals (Прибывшие)	Arr	Absolute number of arrivals
Departures (Выбывшие)	Dep	Absolute number of departures
Migration turnover (Миграционный оборот)	$T = Arr + Dep$	Absolute number of arrivals and departures
Net migration (Миграционный прирост)	$Net = Arr - Dep$	Characterizes migration increase (decrease) of the population

Relative indicators

Indicator	Formula	Interpretation
Arrival rate (Коэффициент прибытия)	$Rate_{arr} = \frac{Arr}{\bar{P}} \cdot 1000$	Number of arrivals per 1000 population
Departure rate (Коэффициент выбытия)	$Rate_{dep} = \frac{Dep}{\bar{P}} \cdot 1000$	Number of departures per 1000 population
Migration turnover rate (Коэффициент миграционного оборота)	$Rate_{turnover} = \frac{Arr + Dep}{\bar{P}} \cdot 1000$ $Rate_{turnover} = Rate_{arr} + Rate_{dep}$	Number of arrivals and departures per 1000 population
Net migration rate (Коэффициент миграционного прироста)	$Rate_{net\ migr} = \frac{Arr - Dep}{\bar{P}} \cdot 1000$ $Rate_{net\ migr} = Rate_{arr} - Rate_{dep}$	Net migration per 1000 population

TESTS

Test 10-1 (select a correct answer)

De-jure population in the region – 100 thou. persons, temporary residents – 20 thou. persons, temporary absent – 10 thou. persons. Calculate de-facto population.

1. 120
2. 110
3. 100
4. 90

Test 10-2 (Match the fertility rate in the left-hand column with its interpretation in the right-hand column):

1	Special birth rate	a	Number of births per year to women of this age group to average annual number of women of this age
2	Total fertility rate	b	Number of births per 1000 population
3	Age-specific fertility rate	c	The sum of age-specific fertility rates calculated for age groups in the interval 15-49 years. This rate shows how many births would give a single woman to children throughout the reproductive period
4	Crude birth rate	d	Number of births per year to women of this age group to average annual number of women of this age

Test 10-3 (Match the mortality and vital indicators in the left-hand column with its interpretation in the right-hand column):

1	Crude death rate	a	Absolute value of difference between births and deaths. Its value may be either positive or negative
2	Natural increase rate	b	Ratio of number of deaths of infants under one year to the number of births
3	Natural increase	c	Characterizes the number of births by the number of deaths
4	Infant mortality rate	d	Number of deaths per 1000 population
5	Pokrovsky's vitality rate	e	Characterizes the increase (decrease) of the population by 1000 people

Test 10-4 (select a correct answer)

The mortality rate is calculated using:

1. population at the beginning of the year
2. population at the end of the year
3. average annual population
4. population in working age

Test 10-5 (select a correct answer):

Determine the missing indicator in the formula for infant mortality rate:

$$\text{Rate}_{\text{infant}} = \frac{\text{Dep}_{\text{under the year}}}{(\quad)} \cdot 1000$$

1. Average population
2. Average number of children
3. Births for the year
4. Total deaths for the year

Test 10-6 (select a correct answer)

Choose the correct formula for migration turnover rate:

1. $\text{Rate}_{arr} = \frac{Arr}{\bar{P}} \cdot 1000$

2. $\text{Rate}_{turnover} = \frac{Arr + Dep}{\bar{P}} \cdot 1000$

3. $\text{Rate}_{net migr} = \frac{Arr - Dep}{\bar{P}} \cdot 1000$

4. $\text{Rate}_{dep} = \frac{Dep}{\bar{P}} \cdot 1000$

Test 10-7 (select a correct answer)

The population of the region at the beginning of the year amounted to 1900 thousand persons, at the end of the year – 2100 thousand persons. During the year 70 thousand persons were born and 80 thousand persons died.

Determine natural increase rate.

1. 5‰

3. 40‰

2. -5‰

4. -40‰

Test 10-8 (select a correct answer)

The population of the region at the beginning of the year amounted to 1900 thousand persons, at the end of the year – 2100 thousand persons. During the year 70 thousand persons were born and 80 thousand persons died.

Determine crude death rate.

1. 5‰

3. 40‰

2. -5‰

4. -40‰

Test 10-9 (select a correct answer)

The population of the region at the beginning of the year amounted to 2000 thousand persons. During the year 100 thousand persons were born and 90 thousand persons died, 20 thousand persons arrived, 30 thousand persons left.

Determine the population at the end of the year.

1. 1900

3. 2100

2. 2000

4. 2200

Test 10-10 (select a correct answer)

The population of the region at the beginning of the year amounted to 2000 thousand persons. During the year 100 thousand persons were born

and 90 thousand persons died, 20 thousand persons arrived, 30 thousand persons left.

Determine the average population.

1. 1900
2. 2000
3. 2100
4. 2200

PRACTICE

10.1. There are the following data on distribution of the population of Russia on age groups (thousand persons):

Age group	Year	
	1959	2010
Under working age	35094	22854
Working age	68609	88370
Over working age	13827	30701
Total:	117530	141925

Perform the following tasks:

1. Determine age structure of population for every year (%).
2. Calculate dependency ratio (child, old-aged and total) (‰).

10.2. There is the distribution of the population by gender and age groups in 2017 (million persons):

Country	Total population	Including		Population at age, years		
		males	females	0-15	16-60	60 and over
Germany	81,2	39,8	41,4	10,7	48,3	22,2
Italy	60,8	29,5	31,3	8,4	35,6	16,8
Russia	146,8	68,0	78,8	25,5	90,8	30,5
France	64,5	31,3	33,2	11,8	36,5	16,2
Indonesia	255,5	128,4	127,1	70,8	162,7	21,6
Republic of Korea	50,6	25,3	25,3	7,0	34,2	9,4
Japan	127,0	61,7	65,2	16,1	68,6	42,3
USA	321,4	158,2	163,2	61,0	193,6	66,8

Source: The Demographic Yearbook of Russia – 2017

Compare the population size and structure of the countries calculating:

- 1) gender structure of population;
- 2) gender disproportion (the number of women per 100 men);
- 3) age structure of population;
- 4) dependency ratio;
- 5) index of aging.

Make a conclusion.

10.3. There are the following demographic data for Krasnodar, 2016 (thousand persons):

Population at the beginning of the year	Births	Deaths	Arrivals	Departures	Population at the end of the year
5514	73,0	71,6	213,9	157,2	

Source: The Demographic Yearbook of Russia – 2017

Determine:

1. Population at the end of the year and average annual population.
2. Crude birth rate, crude death rate and natural increase rate.
3. Absolute and relative migration indicators.

Make a conclusion about the demographic situation in Krasnodar in 2016.

10.4. In 2010 in Russia died 2028 thousand people, and in Israel – 119 thousand.

Determine in which country the death rate is higher if it is known that the average population of Russia – 142,8 million persons and of Israel – 7,6 million persons.

10.5. There are the following data on the dynamics of the number of births and population in Russia:

Indicator	2008	2016
Births, thousand persons	1714	1889
Average annual population, thousand persons	141956	146675
Average annual number of women in reproductive age (15-49), thousand persons	38388	35405

Source: The Demographic Yearbook of Russia – 2017

Determine crude and special birth rates for each year.

10.6. The population of Moscow on January 1, 2016 amounted to 12330 thousand persons, and on January 1, 2017 – 12381 thousand persons. In 2016 145264 persons were born and 123778 persons died.

Determine:

1. Crude birth and death rates.
2. Pokrovsky vitality rate.
3. Natural increase rate.

10.7. The death rate in Portugal in 2015 is equal to 10,5‰, and average population – 10,3 million persons. Number of births in 2015 – 84,5 thousand persons.

Determine birth rate and natural increase rate.

10.8. There are data on number of the born children and died children aged under 1 year for the Central and Southern federal districts of the Russian Federation:

Year	Births, persons		Died children aged under 1 year, persons	
	Central Federal District	Southern Federal District	Central Federal District	Southern Federal District
2011	413427	163816	2656	1164
2012	439610	174813	3414	1453
2013	438654	174135	3343	1379
2014	444919	179259	2889	1277

Source: The Demographic Yearbook of Russia – 2017

Compare Federal Districts of the Russian Federation on the level of infant mortality.

10.9. There are the following data on the number of marriages and divorces for different years in Russia:

Indicator	Year				
	1970	1980	1990	2000	2010
Marriages, units	1319227	1464579	1319928	897327	1215066
Divorces, units	396589	580720	559918	627703	639321
Average annual population, thousand persons	130252	138483	147913	145189	142938

Determine:

1. Average absolute growth and average growth coefficient of marriages and divorces.
2. Marriage rates and divorce rates for each year.
3. Stability of marriages for each year.

10.10. Demographic indicators in Austria in 2015 are the following:

- 1) net migration 113,1 thou. persons;
- 2) arrivals 360780 persons;
- 3) mid-year population 8,7 million persons.

Determine relative migration indicators.

10.11. For the countries of the world there are data on natural change of the population in 2010:

Country	Average annual population, mln. persons	Births, thou. persons	Deaths, thou. persons	Crude birth rate, ‰	Crude death rate, ‰	Natural increase rate, ‰
Sweden	9,3	111,6			10,0	
Italy	60,3	605,0	603,0			
Japan		1148			9,0	0
Britain	61,8			13		4
Tajikistan	7,5		37,5	28		
Mexico	109,6			20	5	
Philippines		2397			5	21
Turkmenistan	5,1		22			14

Perform the following tasks:

1. Calculate the missing indicators in the table.
2. Rank the countries of the world by the values of the natural increase rate and make a conclusion about the differences in the demographic situation of the countries of the world.

10.12. The population of the district on January 1, 2018 was 520 thousand people. During the year 7280 people were born, 5720 people died, 2800 people arrived for permanent residence, 1900 people left.

Determine:

1. Crude birth rate, crude death rate, natural increase rate.
2. Net migration rate.
3. Total increase rate.

10.13. Crude birth rate in region A is 14‰ and in region B is 12‰. It is known that the average annual population in region “B” is 4 times more than the average annual population in region “A”.

Determine the crude birth rate for two regions as a whole.

10.14. Given the data for the region for 2018:

- population at the beginning of the year – 1530 thousand persons;
- population at the end of the year – 1540 thousand persons;
- natural increase rate – 4,0‰;
- vitality rate – 1,33;
- number of deaths of infants under one year – 148 persons;
- percent of women aged 15-49 in total population at the beginning of the year – 32%;
- percent of women aged 15-49 in total population at the end of the year – 34%.

Describe the natural and migration movement of the population of the region in 2018 using absolute and relative indicators you know.

10.15. There are data for the region for 2018:

- population (thousand persons): on January 1, 2018 – 980; on April 1 – 982; on January 1, 2019 – 990;
- number of births for the year – 14283 persons;
- number of arrivals for the year – 15662 persons;
- vitality rate – 1,42;
- percent of women in total population – 55,4%;
- percent of women aged 15-49 in women population – 36,0%.

Determine:

1. Number of deaths for the year.
2. Number of departures for the year.
3. Crude birth rate, crude death rate, natural increase rate and net migration rate.
4. Special birth rate.

10.16. The population of the city at the end of the year amounted to 311511 persons. 4495 persons were born, and 3472 persons died for the year. Net migration for the year was 2000 persons. Number of women aged 15-49 years on average for the year was 121487 persons.

Calculate:

1. Crude birth rate and crude death rate.
2. Special birth rate.
3. Natural increase rate, net migration rate and total increase rate.

10.17. In 2018 the mid-point population of the district center was 240 thousand people. During the year 2880 people were born and 2400 people died. Percent of women aged 15-49 years in population was 33%.

Determine:

1. Crude birth rate and crude death rate.
2. Special birth rate.
3. Vitality rate.



Labour Market Statistics

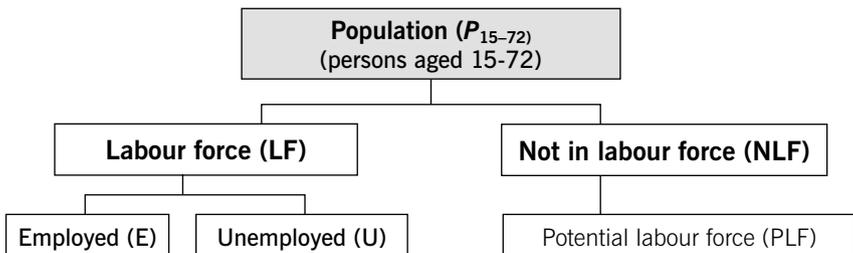
Learning objectives:

- Labour force: employed and unemployed
- Not in labour force including potential labour force
- Duration of unemployment
- Employment rate, labour force participation rate
- Unemployment rate, registered unemployment rate
- Aggregate indicator of unemployment and potential labour force

THEORY

Data on labour force, employment and unemployment are based on the results of labour force sample surveys (population employment surveys) of people aged 15-72.

The survey is conducted in all constituent entities of the Russian Federation on the sampling method basis, followed by extending the results to the entire age-specific population.



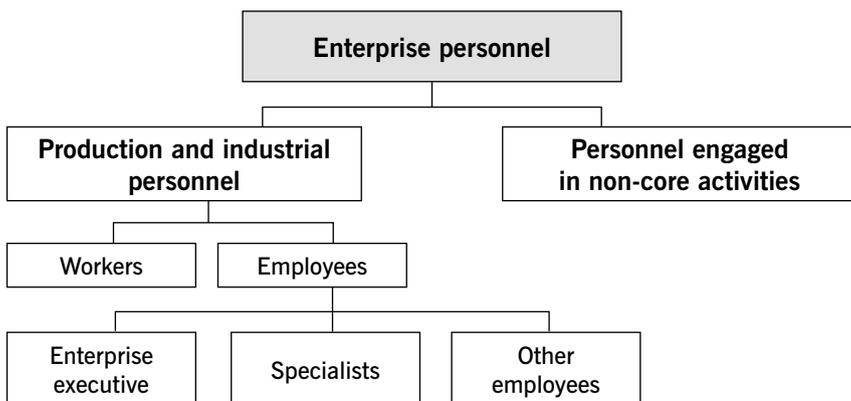
Basic Definitions

Indicators	Russian equivalent	Definition
Labour force (LF)	Рабочая сила	persons aged 15-72, who are considered employed or unemployed during the reference period (the week surveyed)
Employed (E)	Занятые	persons aged 15-72, who, during the week surveyed were engaged (an hour per week at least) in any type of activity to produce goods or provide services for pay or profit. Likewise, the number of employed includes persons temporarily absent from work for a short period of time who retain job attachment during the absence
Unemployed (U)	Безработные	(according to the standards of the International Labour Organization (ILO) are persons aged 15-72 who, during the reference period, equally met the following criteria: <ol style="list-style-type: none"> 1. had no job (gainful activities) 2. had been seeking for a job for four weeks preceding the reference week using any method applicable 3. were ready to take on a job during the reference week. Students, retired and disabled persons are referred to as unemployed, if they had no job, had been seeking for a job and were ready to start working.
Unemployed registered in the state employment service (RU)	Безработные, зарегистрированные в службе занятости	include persons having no job or earnings (labour income), living in the territory of the Russian Federation, registered with the state employment services in the place of residence to find a proper job, seeking for a job and ready to start working.
Potential labour force (PLF)	Потенциальная рабочая сила	unemployed persons who express interest in getting a job for pay or profit, however their current situation restricts their active job search or their readiness to take on a job.
Duration of unemployment	Продолжительность безработицы	the time during which an unemployed person is seeking for a job.

Labour market indicators

Indicator	Formula	Interpretation
Population (P_{15-72}) (persons aged 15-72)	$P = LF + NLF$	Labour force and not in labour force
Labour force (LF)	$LF = E + U$	Employed and unemployed
Labour force participation rate	$LF_Rate = \frac{LF}{P} \cdot 100\%$	Ratio of the labour force to the total population
Employment rate	$LF_Rate = \frac{E}{P} \cdot 100\%$	Ratio of employed population to the total population
Unemployment rate	$LF_Rate = \frac{U}{LF} \cdot 100\%$	Ratio of unemployed to the labour force
Registered unemployment rate	$LF_Rate = \frac{RU}{LF} \cdot 100\%$	Ratio of the unemployed registered in the state employment services to the labour force
Aggregate indicator of unemployment and potential labour force	$AU_Rate = \frac{U + PLF}{LF + PLF} \cdot 100\%$	Ratio of the unemployed and potential labour force to the enhanced labour force concept. The enhanced labour force concept covers the employed, the unemployed and the potential labour force

Enterprise personnel



Notations

Parameter	Russian equivalent	Indicator
H	Списочная численность	Headcount
A	Явочная численность	Attendance
$MD_{attendance}$	Человеко-дни явок	Man-days of attendance
$MD_{absence}$	Человеко-дни неявок	Man-days of absence
$MD_{holidays/weekends}$	Человеко-дни праздничные и выходные	Man-days of holidays and weekends
$MD_{vacation}$	Человеко-дни отпусков	Man-days of vacation
MD_{worked}	Человеко-дни отработанные	Man-days worked
$MD_{downtime}$	Человеко-дни целодневного простоя	Man-days downtime
MH_{worked}	Человеко-часы отработанные	Man-hours worked
AD_{day}	Средняя продолжительность рабочего дня	Average duration of working day
AD_{period}	Средняя продолжительность рабочего периода	Average duration of working period
D	Число календарных дней в периоде	Number of days
WD	Число рабочих дней в периоде	Number of working days
LP	Производительность труда	Labour productivity

Average headcount and average attendance

Indicator	Russian equivalent	Formula
Average headcount	Средняя списочная численность	$\bar{H} = \frac{\sum H}{D}$
		$\bar{H} = \frac{\sum MD_{attendance} + \sum MD_{absence}}{D}$
		$\sum MD_{attendance} = \sum MD_{worked} + \sum MD_{downtime}$
Average attendance	Средняя явочная численность	$\bar{A} = \frac{\sum MD_{attendance}}{WD}$

Working time indicators

Indicator	Russian equivalent	Formula
Average actual duration of working period	Средняя фактическая продолжительность рабочего периода	$\bar{T}_{actual} = \frac{\sum MD_{worked}}{\bar{H}}$
Average plan duration of working period	Средняя плановая продолжительность рабочего периода	\bar{T}_{plan}
Ratio of working period	Коэффициент использования рабочего периода	$K_{\bar{T}} = \frac{\bar{T}_{actual}}{\bar{T}_{plan}} \cdot 100\%$
Average actual duration of working day	Средняя фактическая продолжительность рабочего дня	$\bar{t}_{actual} = \frac{\sum MH_{worked}}{\sum MD_{worked}}$
Average plan duration of working day	Средняя плановая продолжительность рабочего дня	\bar{t}_{plan}
Ratio of working day	Коэффициент использования рабочего дня	$K_{\bar{t}} = \frac{\bar{t}_{actual}}{\bar{t}_{plan}} \cdot 100\%$

Indicators of labour productivity

Labour productivity		
Hour	Day	Month
$LP_{hour} = \frac{PV}{MH_{worked}}$	$LP_{day} = \frac{PV}{MD_{worked}}$	$LP_{month} = \frac{PV}{H}$

PV – production volume

TESTS

Test 11-1 (Match the indicator in the left-hand column with its definition in the right-hand column):

1	Unemployed	a	persons aged 15-72, who are considered employed or unemployed during the reference period (the week surveyed)
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2	Employed	b	unemployed persons who express interest in getting a job for pay or profit, however their current situation restricts their active job search or their readiness to take on a job
3	Potential labour force	c	persons aged 15-72 who, during the reference period, equally met the following criteria: 1. had no job (gainful activities) 2. had been seeking for a job for four weeks preceding the reference week using any method applicable 3. were ready to take on a job during the reference week
4	Labour force	d	persons aged 15-72, who, during the week surveyed were engaged (an hour per week at least) in any type of activity to produce goods or provide services for pay or profit.

Test 11-2 (select correct answers)

Unemployed are persons aged 15-72 who:

- 1) were ready to take on a job during the reference week
- 2) had no job (gainful activities)
- 3) had been seeking for a job were ready to take on a job during the reference week.
- 4) had a job (gainful activities)
- 5) had not been seeking for a job
- 6) weren't ready to take on a job during the reference week

Test 11-3 (select a correct answer)

Potential labour force are:

- 1) persons aged 15-72, who, during the week surveyed were engaged (an hour per week at least) in any type of activity to produce goods or provide services for pay or profit
- 2) persons having no job or earnings (labour income), living in the territory of the Russian Federation, registered with the state employment services
- 3) unemployed persons who express interest in getting a job for pay or profit, however their current situation restricts their active job search or their readiness to take on a job
- 4) persons aged 15-72 who, during the reference period, equally met the following criteria: had no job (gainful activities), had been seeking for a job, were ready to take on a job

Test 11-4 (select a correct answer)

Labor force = 76000 persons, persons not included in labour force = 33000, employed = 72300 persons. Determine unemployment rate.

- | | |
|---------|---------|
| 1. 4,9% | 3. 5,0% |
| 2. 3,4% | 4. 5,1% |

Test 11-5 (select a correct answer)

Labor force = 77000 persons, persons not included in labour force = 23000, unemployed = 4000 persons. Determine employment rate.

- | | |
|----------|----------|
| 1. 94,8% | 3. 73,0% |
| 2. 95,0% | 4. 65,0% |

Test 11-6 (select a correct answer)

Employed = 71000 persons, unemployed = 5000 persons, persons not included in labour force = 24000. Determine labour force participation rate.

- | | |
|----------|----------|
| 1. 71,0% | 3. 67,0% |
| 2. 76,0% | 4. 61,0% |

Test 11-7 (select a correct answer)

The number of unemployed at the beginning of the month is the following (persons):

January – 5000, February – 5100, March – 5150, April – 5200.

Determine average number of unemployed for the first quarter.

- | | |
|---------|---------|
| 1. 5083 | 3. 5117 |
| 2. 5113 | 4. 5120 |

Test 11-8 (select a correct answer)

The aggregate indicator of unemployment and potential labour force is:

1. Ratio of unemployed to the labour force
2. Ratio of the unemployed registered in the state employment services to the labour force
3. Ratio of the unemployed and potential labour force to the enhanced labour force concept
4. Ratio of employed population to the total population
5. Ratio of the labour force to the total population

Test 11-9 (select a correct answer)

The labour force participation rate is:

1. Ratio of unemployed to the labour force
2. Ratio of the unemployed registered in the state employment services to the labour force

3. Ratio of the unemployed and potential labour force to the enhanced labour force concept
4. Ratio of employed population to the total population
5. Ratio of the labour force to the total population

Test 11-10 (select a correct answer)

The registered unemployment rate is:

1. Ratio of unemployed to the labour force
2. Ratio of the unemployed registered in the state employment services to the labour force
3. Ratio of the unemployed and potential labour force to the enhanced labour force concept
4. Ratio of employed population to the total population
5. Ratio of the labour force to the total population

PRACTICE

11.1. There are data on labour force and people not in labour force in Russia (thousand persons):

Indicator	2010	2015	2016
Labour force	75 478	76 588	76 636
including unemployment	5 544	4 264	4 243
Not in labour force	36 055	34 187	33 590
including potential labour force	1 731	1 343	1 203
Unemployed registered in the state employment service	1 589	1 001	895

Source: The Demographic Yearbook of Russia – 2017

Determine:

1. Growth rate of employed and unemployed: a) in 2015 compared to 2010; b) in 2016 compared to 2015; c) in 2016 compared to 2010.
2. The average growth coefficient and the average rate of increase (decrease) for the period 2010-2016.
3. Labour force participation rate, employment rate, unemployment rate and registered unemployment rate.
4. Proportion of potential labour force in population not in labour force (%).

11.2. The results of labour force sample survey (population employment survey) in 2017 are the following:

Date	Employed, thousand persons	Unemployed, thousand persons
01.01.2017	71838	4290
01.04.2017	71850	4050
01.07.2017	72523	3907
01.10.2017	72743	3859
01.01.2018	72611	3877

Determine:

1. Average number of employed, unemployed and labour force for 2017 year.
2. Average absolute growth and average growth coefficient of employed and unemployed for the period from 01.01.2017 to 01.01.2018.
3. Unemployment rate for 2017 year.

11.3. Rosstat data on the number of unemployed in 2017 are the following:

	at the beginning of the month						
	January	February	March	April	May	June	July
Number of unemployed, thou. persons	4290	4231	4112	4050	3945	3860	3907

Calculate the average number of unemployed:

- 1) in January;
- 2) in the I quarter;
- 3) in the II quarter;
- 4) in the first half of the year.

11.4. The number of employed men in Russia in 2014 was 36059 thousand and unemployed – 2323 thousand. Labour force participation rate of men was 74,4%.

Determine:

- 1) total population of men;
- 2) population of men not in labour force.

11.5. According to the labour force sample survey, the duration of unemployment in December 2018 amounted to:

Duration of unemployment, months	under 3	3 – 6	6 – 9	9 -12	12 and over
Unemployed, %	34	20	10	9	27

Determine:

1. Average duration of unemployment.
2. Mode and median.
3. Coefficient of variation.

11.6. There are data on the distribution of the employed and unemployed population by the age:

Age, years	Proportion, %	
	Employed	Unemployed
under 20	2	10
20-30	22	30
30-40	24	22
40-50	30	24
50-60	18	12
60 and over	4	2
Total:	100	100

Determine:

1. Average and mode age of employed and unemployed.
2. Skewness of the distribution of the employed and unemployed using Karl Pearson's formula.

11.7. The number of employed and unemployed women aged 20-49 with and without children in Russia in 2017 are the following:

thousand persons

	Employed	Unemployed	Not in labour force
Women with children under 18:			
• 1 child	8336	371	1477
• 2 children	3817	232	1042
• 3 and more children	722	78	449
Women without children under 18	12250	765	2577

Source: The Demographic Yearbook of Russia – 2017

Determine labour force participation rate, employment and unemployment rates for each group of women.

11.8. There are the following main indicators of labour market for Tver region on 2017:

- 1) labour force – 690 thousand persons;
- 2) not in labour force – 296 thousand persons;
- 3) unemployment rate – 5,8%.

Determine the number of employed and employment rate for Tver region.

11.9. There are the following main indicators of labour market for Kaliningrad region in 2017:

- 1) employed – 493 thousand persons;
- 2) unemployment rate – 6%;
- 3) registered unemployment rate – 1,0%.

Determine the number of unemployed and unemployed registered in the state employment service and the difference between them.

11.10. There are data on the unemployment rates in different countries in 2016:

N_e	Country	Unemployment rate, %	N_e	Country	Unemployment rate, %
1	Australia	5,7	12	Japan	3,1
2	Austria	6,0	13	Lithuania	7,9
3	Belgium	7,8	14	Netherlands	6,0
4	Bulgaria	7,6	15	Norway	4,7
5	Canada	7,0	16	Poland	6,2
6	Denmark	6,2	17	Republic of Korea	3,7
7	Finland	8,8	18	Russia	5,5
8	France	10,1	19	Sweden	7,0
9	Germany	4,1	20	Switzerland	4,9
10	Hungary	5,1	21	United Kingdom	4,8
11	Italy	11,7	22	USA	4,9

Group countries by unemployment rate and make three groups.

For each group count the number of the countries and give a list of the countries.

11.11. There are data on main indicators of labour market and ratio of crimes in the Central Federal District of Russia:

Region	Employed, thousand persons	Unemployed, thousand persons	Number of crimes per 10 thousand persons
Belgorod Region	789	33	88
Bryansk Region	596	29	129
Vladimir Region	696	41	137
Voronezh Region	1113	52	147
Ivanovo Region	506	30	140
Kaluga Region	520	23	160
Kostroma Region	310	18	135
Kursk Region	545	24	109
Lipetsk Region	574	24	107
Moscow Region	3863	133	121
Orel Region	360	25	123
Ryazan Region	515	24	84
Smolensk Region	487	31	143
Tambov Region	498	24	116
Tver Region	649	40	161
Tula Region	767	33	80
Yaroslavl Region	632	45	145

Perform the following tasks:

1. Determine unemployment rate for each region.
2. Define Spearman rank correlation coefficient and make a conclusion about the relationship between the unemployment rate and number of crimes.

11.12. Given the data on unemployment rates of leading countries:

Month -year	Unemployment rate, %		
	German	United Kingdom	USA
01-2018	5,4	4,3	4,1
02-2018	5,4	4,2	4,1
03-2018	5,3	4,2	4,1

Окончание

Month -year	Unemployment rate, %		
	German	United Kingdom	USA
04-2018	5,3	4,2	4,1
05-2018	5,2	4,2	3,9
06-2018	5,2	4,0	3,8
07-2018	5,2	4,0	4,0
08-2018	5,2	4,0	3,9
09-2018	5,1	4,1	3,7

Source: Federal Statistical Office

Source: Office for National Statistics

Source: Federal Reserve, Department of Commerce, Department of Labor

Perform the following tasks:

1. Determine average growth rate and compare the dynamics of the unemployment rates of leading countries.
2. Make a forecast of unemployment rates for 10-2018 using the trend line equation.
3. Determine mean absolute percentage error (MAPE) and make a conclusion about the accuracy of equation.

11.13. There are data on main indicators of labour market by constituent entities of the Russian Federation in 2016, million persons (officially registered unemployed- thousand persons):

Federal District	Population		Number of employed	Number of unemployed	Working age population	Officially registered unemployed
	total	including at the age of 15-72 years				
Central	39,3	29,7	20,5	0,8	22,3	163
North West	13,9	10,7	7,3	0,3	7,9	70
South	16,4	12,1	7,6	0,5	9,2	67
North-Caucasian	9,8	6,9	4,0	0,5	5,7	144
Privolzhsky (Volga)	29,6	22,3	14,7	0,7	16,6	157
Ural	12,3	9,3	6,1	0,4	7,0	85

Окончание

Federal District	Population		Number of employed	Number of unemployed	Working age population	Officially registered unemployed
	total	including at the age of 15-72 years				
Siberian F	19,3	14,4	9,0	0,8	10,9	154
Far East	6,2	4,8	3,2	0,2	3,6	54
Russian Federation	146,8	110,2	72,4	4,2	83,2	894

Source: The Russian Statistical Yearbook, 2017

Perform the following tasks:

1. Determine for Russian Federation and each constituent entities:
 - a) labour force;
 - b) number of persons not included in labor force persons
 - c) labour force participation rate;
 - d) employment rate;
 - e) unemployment rate;
 - f) registered unemployment rate;
 - g) employment rate for the whole population, working age population.
2. Compare the indicators calculated for constituent entities.

11.14. Given the data on the number of employees in March (persons):

Day of the month	Headcount	Attendance (number of persons at work)
from 1 to 2	200	195
from 5 to 7	202	198
9	203	203
from 12 to 16	210	210
from 19 to 21	212	200
from 22 to 23	208	205
from 26 to 30	210	204

Holidays and weekends: 3, 4, 8, 10, 11, 17, 18, 24, 25, 31.

Determine:

- 1) average headcount;
- 2) average attendance (number of persons at work).

11.15. The company started its work on February 7.

The headcount in February was the following:

7-8 – 80 persons; 11-15 – 150 persons; 18-22 – 160 persons; 25-28 – 165 persons.

Holidays and weekends in February: 2, 3, 9, 10, 16, 17, 23, 24.

Average headcount in March was 162 persons, in II quarter – 170 persons, in III quarter – 164 persons, in October – 165 persons, in November – 170 persons, in December – 166 persons.

Calculate average headcount for:

- 1) February;
- 2) I quarter;
- 3) I half of the year;
- 4) IV quarter;
- 5) II half of the year;
- 6) year.

11.16. There are data on the company for October:

Indicator	Value
Man-days worked	3870
Man-days downtime	20
Man-days of absence in working days	260
Man-days of absence in weekends	1440
Number of working days	23

Determine average headcount and average attendance (number of persons at work) in October.

11.17. There are data on the company for April:

- man-days worked – 8474;
- man-days downtime – 10;
- man-days of absence – 4116,
 - including man-days of holidays and weekends – 3780;
- man-days of vacation – 200.

Man-hours worked for the month – 63630. There are 21 working days in April. Average plan duration of working day – 8 hours.

Determine:

- 1) average headcount and average attendance (number of persons at work);
- 2) average actual duration of working period;

- 3) average actual duration of working day;
- 4) ratio of working period;
- 5) ratio of working day.

11.18. Given the following data:

Indicator	March	April
Production volume, thousand pieces	100	120
Average headcount of personnel,	360	345
including workers	317	304
Man-days worked by workers	6660	6316
Man-hours worked by workers	50000	48000

Perform the following task:

1. Determine for workers for each month:
 - a) hour labour productivity;
 - b) day labour productivity;
 - c) month labour productivity.
2. Determine personnel month labour productivity for each month.
3. Analyze the change of labour productivity in April compared to March.
4. Absolute change of production volume in April compared to March due to:
 - a) change of man-hours worked by workers;
 - b) change of labour productivity;
5. Determine for each month time spent per unit of output (labor intensity of production);
6. Analyze the change of time spent per unit of output;
7. Determine the change in the Fund of time worked in April compared to March total, including due to each factor:
 - a) changes in production volume;
 - b) changes in labour productivity.



UNIT 12

Statistics of living standard of population

Learning objectives:

- Employee wage
- Average nominal and real wage
- Nominal wage index and real wage index
- Index of average wage
- Types of income
- Distribution of population by average per capita money income
- Median per capita money income
- Modal per capita money income
- Coefficient of funds

THEORY

Employee wages

Employee wages include:

- wages paid in cash and noncash form, accrued by an organization to employees for time worked and not worked
- compensation payments related to working conditions and work schedule
- supplemental and additional payments
- bonuses
- one-time incentive payments.

Average nominal wages are wages including taxes and other withholdings in accordance with the legislation of the Russian Federation, expressed in average monetary units per employee.

$$\overline{W}_{nominal} = \frac{Wage\ fund}{Employed}$$

The nominal wage index is based on current survey data as the ratio of average monthly nominal accrued wages in the current period to the average monthly nominal accrued wages in the base period

$$I_{\overline{W}_{nominal}1/0} = \frac{\overline{W}_{nominal\ 1}}{\overline{W}_{nominal\ 0}}$$

The real wage index is computed as the ratio of the nominal wage index to the consumer price index for the same period.

$$I_{\overline{W}_{real}1/0} = \frac{I_{\overline{W}_{nominal}\ 1/0}}{I_{consumer\ price\ 1/0}}$$

The real wages are computed as the ratio of the nominal wage the consumer price index for the same period.

$$\overline{W}_{real1/0} = \frac{\overline{W}_{nominal}}{I_{consumer\ price\ 1/0}}$$

Index of average wages

$$I_{\overline{W}1/0} = \frac{\sum \overline{W}_1 E_1}{\sum E_1} : \frac{\sum \overline{W}_0 E_0}{\sum E_0}$$

\overline{W}_0 – average monthly wages in base period

\overline{W}_1 – average monthly wages in current period

E_0 – employed in base period

E_1 – employed in current period

Absolute change of average wages:

$$\Delta \overline{W} = \frac{\sum \overline{W}_1 E_1}{\sum E_1} - \frac{\sum \overline{W}_0 E_0}{\sum E_0}$$

Index of change of average wages due to chance of wages w

$$I_{\overline{W}1/0}^w = \frac{\sum \overline{W}_1 E_1}{\sum E_1} : \frac{\sum \overline{W}_0 E_1}{\sum E_1}$$

Absolute change of average wages due to chance of wages w

$$\Delta \overline{W}^w = \frac{\sum \overline{W}_1 E_1}{\sum E_1} - \frac{\sum \overline{W}_0 E_1}{\sum E_1}$$

Index of change of average wage due to chance of the structure of employed E

$$I_{W_{1/0}}^E = \frac{\sum \bar{W}_0 E_1}{\sum E_1} : \frac{\sum \bar{W}_0 E_0}{\sum E_0}$$

Absolute change of average wage due to chance of the structure of employed E

$$\Delta_W^E = \frac{\sum \bar{W}_0 E_1}{\sum E_1} - \frac{\sum \bar{W}_0 E_0}{\sum E_0}$$

Types of income

Income	Russian equivalent	Characteristics
Gross disposable household income	Валовой располагаемый доход домашних хозяйств	The amount of income derived from the primary distribution (wages and salaries, mixed incomes, net property incomes), and as the result of redistribution of income in a monetary form (current transfers)
Money income of population	Денежные доходы населения	Includes income of persons engaged in entrepreneurial activities, paid wages of employees (accrued remuneration, adjusted for the change in arrears), social benefits (pensions, benefits, scholarships, insurance reimbursement and other transfers), property income in the form of interest on deposits, securities, dividends and other income
Real money income	Реальные денежные доходы	Calculated by dividing of nominal value of money income of population by consumer price index $Income_{real} = \frac{Income_{nominal}}{I_{consumer\ price}}$
Real money income index	Индекс реальных денежных доходов	Relative indicator, which is calculated by dividing of index of nominal value of money income by consumer price index $I_{Income_{real1/0}} = \frac{I_{Income\ nominal_{1/0}}}{I_{consumer\ price_{1/0}}}$
Average per capita money income	Среднедушевые денежные доходы (в месяц)	Calculated by dividing annual money income by 12 and by midyear population

**Volume and structure of money
income of population**

Money income	Volume, billion rubles	Money income structure, %
Income from entrepreneurial activities		
Labour remuneration		
Social payments		
Property income		
Other income		
Total:		100,0

Distribution of population by average per capita money income is disparity of population in terms of material wealth and is an indicator of number (or shares) of population, grouped in specified range by level of average per capita money income.

Median per capita money income (median) is defined as income level for which one half of population has per capita money income below the median, the other half of population has per capita money income above the median.

$$M_e = x_{Me} + i \cdot \frac{N_{Me} - Cf_{Me-1}}{f_{Me}}$$

x_{Me} – lower level of median interval

i – width of median interval

f_{Me} – frequency of median interval

Cf_{Me-1} – cumulative frequency of the interval before median interval

N_{Me} – number of the median $N_{Me} = \frac{\sum f}{2}$

Modal per capita money income is value of per capita money income, most commonly occurring in the population distribution by average per capita money income.

$$M_o = x_{Mo} + i \cdot \frac{f_{Mo} - f_{Mo-1}}{(f_{Mo} - f_{Mo-1}) + (f_{Mo} - f_{Mo+1})}$$

x_{Me} – lower level of modal interval

i – width of modal interval

f_{Mo} – frequency of modal interval

f_{Mo-1} – frequency of interval before modal interval

f_{Mo+1} – frequency of interval before modal interval

Coefficient of funds (income difference's coefficient) is the degree of social stratification and is defined as a ratio between the average levels of money income of 10 percent of population with the highest income and 10 percent of population with the lowest income.

$$\text{Coefficient of funds} = \frac{D_9}{D_1}$$

$$D = x_D + i \cdot \frac{N_D - Cf_{D-1}}{f_D}$$

x_D – lower level of decile interval

i – width of modal interval

f_D – frequency of decile interval

Cf_{D-1} – cumulative frequency of the interval before decile interval

N_D – number of decile $N_D = \frac{k \sum f}{10} \quad k=1,2,3,\dots,9$

TESTS

Test 12-1 (select a correct answer)

The real wage index is computed as:

- 1) the ratio of the nominal wage index to the consumer price index for the same period
- 2) the ratio of the real wage index to the consumer price index for the same period
- 3) the ratio of the nominal wage index to the consumer price index for the previous period
- 4) the ratio of the nominal wage index to the real wage index

Test 12-2 (select a correct answer)

The real wages is computed as

- 1) the ratio of the nominal wages the real wage index for the same period
- 2) the ratio of the nominal wages the consumer price index for the same period
- 3) the ratio of the nominal wages the consumer price index for the previous period
- 4) the ratio of the nominal wages the nominal wage index for the same period

Test 12-3 (Match the type of income in the left-hand column with its characteristics in the right-hand column):

1	Real money income	a	Calculated by dividing annual money income by 12 and by midyear population
2	Money income of population	b	The amount of income derived from the primary distribution (wages and salaries, mixed incomes, net property incomes), and as the result of redistribution of income in a monetary form (current transfers)
3	Gross disposable household income	c	Calculated by dividing of nominal value of money income of population by consumer price index
4	Average per capita money income	d	Includes income of persons engaged in entrepreneurial activities, paid wages of employees (accrued remuneration, adjusted for the change in arrears), social benefits (pensions, benefits, scholarships, insurance reimbursement and other transfers), property income in the form of interest on deposits, securities, dividends and other income

Test 12-4 (select a correct answer)

Nominal wage amounted to 38 600 rubles in January and 38 800 rubles in February. Consumer price index in February compared to January amounted to 105%.

Determine the real wage in February.

- | | |
|----------|----------|
| 1. 40530 | 3. 36952 |
| 2. 40740 | 4. 36762 |

Test 12-5 (select a correct answer)

Given the real money income index:

	I Quarter	II Quarter	III Quarter	IV Quarter
Real money income index, % to the previous quarter	90	108	110	115

Determine real money income index in the IV quarter compared to the I quarter.

- 136,6%
- 123,0%
- 126,5%
- 130,5%

Test 12-6 (select a correct answer)

Given the distribution of population by per capita money income:

Per capita money income, rubles per month	Population, %
up to 7000	6
7000 – 10000	8
10000-14000	13
14000-19000	15
19000-27000	19
27000-45000	22
45000-60000	8
60000 and more	9
Total:	100

Define median per capita money income interval.

- | | |
|----------------|--------------------|
| 1. 14000-19000 | 3. 27000-45000 |
| 2. 19000-27000 | 4. 60000 and more. |

Test 12-7 (select a correct answer)

Given the distribution of population by per capita money income:

Per capita money income, rubles per month	Population, %
up to 7000	6
7000 – 10000	8
10000-14000	13
14000-19000	15
19000-27000	19
27000-45000	22
45000-60000	8
60000 and more	9
Total:	100

Define the ninth decile interval.

- | | |
|----------------|--------------------|
| 1. 19000-27000 | 3. 45000-60000 |
| 2. 27000-45000 | 4. 60000 and more. |

Test 12-8 (select a correct answer)

Choose the correct formula for the coefficient of funds:

1. $\frac{D_{10}}{D_1}$
2. $\frac{D_9}{D_0}$
3. $\frac{D_9}{D_1}$
4. $\frac{D_{10}}{D_0}$

Test 12-9 (select a correct answer)

Choose the correct formula for absolute change of average wage due to change of the structure of employed:

1. $\frac{\sum \bar{W}_1 E_1}{\sum E_1} - \frac{\sum \bar{W}_0 E_0}{\sum E_0}$
2. $\frac{\sum \bar{W}_1 E_1}{\sum E_1} - \frac{\sum \bar{W}_0 E_1}{\sum E_1}$
3. $\frac{\sum \bar{W}_0 E_1}{\sum E_1} - \frac{\sum \bar{W}_0 E_0}{\sum E_0}$
4. $\frac{\sum \bar{W}_0 E_0}{\sum E_0} - \frac{\sum \bar{W}_0 E_1}{\sum E_1}$

Test 12-10 (select a correct answer)

Income level for which one half of population has per capita money income below it, the other half of population has per capita money income above it, called

1. Modal per capita money income
2. Median per capita money income
3. Real money income
4. Nominal money income.

PRACTICE

12.1. Given the following data on accrued average monthly wages of employees:

Year	Accrued average monthly wages	
	rubles	US dollars (based on annual average official US dollar exchange rate)
2012	26629	857
2013	29792	936

Окончание

Year	Accrued average monthly wages	
	rubles	US dollars (based on annual average official US dollar exchange rate)
2014	32495	856
2015	34030	561
2016	36709	549

Calculate:

1. Absolute growth and rate of increase with variable base of average monthly wages in rubles and in dollars.

2. Average absolute growth and rate of increase of average monthly wages in rubles and in dollars.

12.2. The following data are available for 3 years for Russian Federation:

Indicator	2015	2016	2017
Average nominal wages, rubles	34 030	36 709	39 167
Consumer price index, % to the previous year	112,9	105,4	102,5

Source: Short-Term Economic Indicators of the Russian Federation, 2018

Perform the following tasks:

1. Calculate the real average wages for each year (for the comparison base take 2015 year).

2. Determine average growth rate of nominal and real wages for the period 2015-2017.

12.3. For the Russian Federation data on wages dynamics are available:

Indicator	2014	2015	2016	2017
Nominal wage index, % to the previous year	109,1	105,1	107,9	106,7
Real wage index, % to the previous year	101,2	91,0	100,8	102,9

Source: Short-Term Economic Indicators of the Russian Federation, 2018

Determine growth coefficient of nominal and real wage:

- 1) in 2017 compared to 2013 year;
- 2) average for the period 2013-2017.

12.4. There are the following data on the dynamics of the average monthly wage of employees:

Region	Average nominal wages, rubles		Employed, thousand persons		Consumer price index, % in 2016
	2012	2016	2012	2016	compared to 2012
Smolensk region	17 942	25 097	491	487	142,1
Tula region	20 397	29 402	766	767	143,7

Source: Russian statistical yearbook. 2017

Determine:

1. The real average wages for each region and its dynamic in 2016 compared to 2012.

2. Index of average real wages (%) and absolute change of average wage for both regions.

3. Index of change of average wages due to chance of wages and absolute change of average wages due to chance of wages.

4. Index of change of average wages due to chance of the structure of employed and absolute change of average wages due to chance of the structure of employed.

12.5. Given the following data:

Quarter	The nominal wage index, % to the previous quarter	Consumer price index, % to the previous quarter
I	102,5	100,5
II	101,4	103,6
III	103,4	104,1
IV	102,6	103,0

Source: Short-Term Economic Indicators of the Russian Federation, 2018

Determine dynamic of real wage:

1. from quarter to quarter;

2. for the whole period;

3. on average quarterly.

12.6. There are data on the wages of employees for 2016:

Federal district	January		December	
	Wage fund, million rubles	Average nominal wages, thousand rubles	Wage fund, million rubles	Employed, thousand persons
Central	991 950	38,9	1 021 080	25 527
North-West	269 280	37,4	275 766	7 257
Privolzhsky	376 320	25,6	388 344	14 710

Source: Russian statistical yearbook. 2017

Determine:

1. Index of average wage (%) and absolute change of average wage.
2. Index of change of average wage due to chance of wages and absolute change of average wage due to chance of wages.
3. Index of change of average wage due to chance of the structure of employed and absolute change of average wage due to chance of the structure of employed.

12.7. Given the data on the level of wages in Moscow and St. Petersburg and the dynamics of consumer prices:

Region	Average nominal wage, rubles		Consumer price index, % in 2015 to 2014
	2014	2015	
Moscow	55 323	58 270	114,2
St. Petersburg	38 590	42 178	113,2

Source: Russian statistical yearbook. 2017

Determine nominal wage indices and real wage indices for Moscow and St. Petersburg.

12.8. There are the following data for October 2018 on the wages of employed in the economy, depending on the length of service:

Length of service, years	Employed, million persons	Average wage, thousand rubles	Variation within groups
Up to 5	9,5	28,4	0,20
5 – 10	4,1	30,0	0,45
10 and more	6,6	30,1	0,60

Define:

1. Variation between groups.
2. Average of the variation within groups.
3. Total variation.
4. Correlation ratio between length of service and average wage.

12.9. Given the data on the distribution of employees by average monthly wage:

Average monthly wage, thousand rubles	Manages	Specialists	Other employees	Workers
20–30	–	–	4	–
30–40	–	–	5	23

Окончание

Average monthly wage, thousand rubles	Manages	Specialists	Other employees	Workers
40–50	–	2	18	65
50–60	–	8	11	77
60–70	–	14	2	69
70–80	–	25	2	77
80–90	–	20	–	32
90–100	–	8	–	–
100–110	1	3	–	–
110–120	3	–	–	–
120–130	3	–	–	–
130–140	5	–	–	–
140–150	4	–	–	–
150 and more	2	–	–	–

Calculate for each category of personnel:

1. Average wages.
2. Modal and median wages.

12.10. Given the data on length of service and the average monthly wage of 10 employees of the firm:

Average monthly wage and length of service of the staff

№	Length of service, years	Average monthly wage, thousand rubles
1	2	21
2	2	22
3	3	26
4	4	30
5	5	33
6	5	30
7	5	37
8	6	36
9	6	39
10	7	39

Perform the following tasks:

1. Define Karl Pearson's coefficient of correlation.
2. Determine the equation of regression.
3. Determine the predicted value of average monthly wage for length of service 8 years.

12.11. Given the data on volume of money income of Russian population:

Money income	Volume, billion rubles	
	2010	2015
Income from entrepreneurial activities	2873	4245
Labour remuneration	21190	35105
Social payments	5762	9768
Property income	2023	3340
Other income	650	1068
Total:	32498	53526

Source: Russian statistical yearbook. 2017

Perform the following tasks:

1. Determine the structure of money income for 2010 and 2015 (in % with accuracy up to 0.1).
2. Calculate average growth coefficient for the period 2010-2015.

12.12. Given the data on the structure of money income of Russian population:

Money income	Share of population, %	
	2008	2016
Income from entrepreneurial activities	10,2	7,8
Labour remuneration	68,4	64,7
Social payments	13,2	19,2
Property income	6,2	6,3
Other income	2,0	2,0
Total:	100,0	100,0

Source: Russian statistical yearbook. 2017

It is known that the total amount of income in 2008 amounted to 25244 billion rubles, and in 2016 – 54103 billion rubles.

Define:

1. The absolute value of income for each source of income for each year.
2. The growth rate of the amount of income for each source of income in 2016 compared to 2016.

12.13. Per capita nominal income of the population of Russia in January 2015 amounted to

20 630 RUB., February 2015 – 27 680 RUB. Consumer price index in February compared to January amounted to 102,2%.

Determine:

- 1) the real income per capita in February;
- 2) absolute growth of the real income per capita.

12.14. Given the real money income index in 2016, in % to the previous quarter:

	I Quarter	II Quarter	III Quarter	IV Quarter
Real money income index, % to the previous quarter	73,5	112,3	99,9	116,5

Source: Russian statistical yearbook. 2017

Determine growth coefficient:

- 1) in the IV Quarter 2016 compared to the IV Quarter 2015;
- 2) average for the period from the IV Quarter 2015 to the IV Quarter 2016.

12.15. Given the following data:

Quarter	The nominal money income index, % to the previous quarter	Consumer price index, % to the previous quarter
I	101,7	100,8
II	101,8	102,4
III	102,4	105,6
IV	103,1	104,8

Source: Russian statistical yearbook. 2017

Determine dynamic of real money income:

- 1) from quarter to quarter;
- 2) for the whole period;
- 3) on average quarterly.

12.16. Given the distribution of population of Russia by per capita money income, based on data from household budget sample surveys and macroeconomic indicator average per capita money income of population, 2018:

Per capita money income, rubles per month	Population, %
up to 7000	5,9
7000 – 10000	8,2
10000-14000	12,8
14000-19000	15,2
19000-27000	18,9
27000-45000	22,6
45000-60000	7,9
60000 and more	8,5
Total:	100

Source: Russian statistical yearbook. 2017

Define:

1. Median income per capita.
2. The 1-st and the 9-th deciles and the decile ratio (coefficient of funds).

12.17. The following data are available for 3 years for Russian Federation:

Indicator	2014	2015	2016
Nominal average per capita money income, rubles	27 766	30 467	30 744
Consumer price index, % to the previous year	112,9	105,4	102,5

Perform the following tasks:

1. Calculate the real average per capita money income for each year (for the comparison base take 2014 year).
2. Determine average growth coefficient of nominal and real average per capita money income for the period 2014-2016.

12.18. Given the data on dynamic of average per capita money income:

	Years					
	2012	2013	2014	2015	2016	2017
Average per capita money income, thousand rubles	23	26	28	30	31	31

Make a forecast of the volume of the average per capita money income for 2018:

- 1) using the average growth coefficient;
- 2) using the trend line equation.

12.19. Given the data on household income distribution in the United States in 2017:

Annual household income in thousand U.S. dollars	Percentage of U.S. households
under 15	11,2
15 – 25	9,6
25 – 35	9,4
35 – 50	12,9
50 – 75	17,0
75 – 100	12,3
100 – 150	14,0
150 – 200	6,6
200 and over	7,0
Total:	100,0

Source: © Statista 2018

Define coefficient of funds.



UNIT 13

System of National Accounts (SNA)

Learning objectives:

- Gross domestic product
- Three methods of calculating GDP
- Methods of revaluation of GDP to constant prices
- GDP change due to change of prices and quantity

THEORY

The System of National Accounts is based on methodological principles, developed by UN, IMF, World Bank, OECD and Eurostat.

Gross Domestic Product (GDP) is one of the most important indicators of the system of national accounts. It describes the result of production activities of all economic entities – residents of the country. It reflects the value of final goods and services, produced by these units during a reporting period at final customer prices.

Three methods of calculating GDP

Production approach (Производственный метод)	<ul style="list-style-type: none"> sum of gross value added of all industries or sectors at basic prices plus net taxes on products (taxes on products less subsidies on products)
Income approach (Распределительный метод)	<ul style="list-style-type: none"> the sum of primary incomes, paid by units-residents to participating members, is not independent, as in accordance with methodology, not all income indicators can be measured directly
Expenditure approach (Метод конечного использования)	<ul style="list-style-type: none"> sum of expenditures of all institutional sectors for final consumption, gross capital formation and net exports

Production approach

GDP	=	Output	–	Intermediate consumption	+	Taxes on products	–	Subsidies on products
						Net taxes on products		

Indicator	Russian equivalent	Characteristics
Output	Выпуск	Total value of goods and services, resulting from production activity of resident units of economy
Intermediate consumption	Промежуточное потребление	Consists of value of goods and services, transformed in production process during a reporting period
Taxes on products	Налоги на продукты	Taxes, levied in proportion to the quantity or value of goods and services, produced, sold or imported by residents. They include value added tax (VAT), excises, taxes on imported goods and services, etc.
Subsidies on products	Субсидии на продукты	Subsidies paid per unit of goods or services produced, i.e. in proportion to the quantity or value of goods and services produced, sold or imported by resident enterprises

Income approach

GDP	=	Compensation of employees	+	Taxes on production and imports	-	Subsidies on production and imports	+	Gross operating surplus and gross mixed income
				Net taxes on production and imports				

Indicator	Russian equivalent	Characteristics
Compensation of employees	Оплата труда наёмных работников	Remuneration in cash or in kind, paid by employer to employees for work done during reporting period
Taxes on production and imports	Налоги на производство и импорт	Compulsory, non-refundable payments levied by public administration bodies on producing units in connection with production and imports of goods and services or use factors of production. Consist of: – taxes on products – other taxes on production
Subsidies on production and imports	Субсидии на производство и импорт	Current unpaid non-refundable payments made by the state to enterprises in connection with the production, sale or import of goods and services. Consist of: – subsidies on products – other subsidies on production
Gross operating surplus and gross mixed income	Валовая прибыль экономики и валовые смешанные доходы	Part of gross domestic product, which remains with producers after deducting expenses, related to compensation of employees and net taxes on production and imports. It measures profit (or loss), resulting from production, before accounting of payments or receiving of income from property

Expenditure approach

GDP	=	Final consumption expenditures	+	Gross capital formation	+	Balance of export and import	+	Statistical discrepancy
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Indicator	Russian equivalent	Characteristics
Final consumption expenditures	Расходы на конечное потребление	Consists of final consumption expenditures of: -households -government -non-profit institutions serving households
Gross capital formation	Валовое накопление	Includes: -gross fixed capital formation -changes in inventories -acquisitions less disposals of non-produced non-financial assets
Balance of export and import	Сальдо экпорта и импорта	Calculated as the difference between exports and imports of goods and services
Statistical discrepancy	Статистическое расхождение	Indicator, which characterizes error in construction of all accounts. Statistical discrepancy, which does not exceed 4-5% of GDP, is evidence of satisfactory quality of calculation

Methods of revaluation of GDP to constant prices

Revaluation of GDP to constant prices (base period prices) is used to eliminate the influence of inflation on GDP dynamics.

Notation	Interpretation
$q_1 p_1$	current GDP at current prices (nominal GDP)
$q_0 p_0$	base GDP at base prices
$q_1 p_0$	current GDP at base prices (real GDP)
$I_{q\ GDP}$	GDP quantity index
$I_{p\ GDP}$	GDP price index (GDP deflator index)
$I_{pq\ GDP}$	GDP value index

1. Deflation method

Based on price indices

Paasches price index	GDP deflator index
$I_{p\ 1/0} = \frac{\sum p_1 q_1}{\sum p_0 q_1}$	$I_{deflator\ GDP} = \frac{GDP_{nominal}}{GDP_{real}}$

GDP at comparable prices is equal to GDP at current prices divided by the deflator of the corresponding period:

$$\sum q_1 p_1 : I_{p1/0} = \sum q_1 p_0$$

2. Extrapolation method

Based on quantity indices

$$I_{q\text{ GDP}} = \frac{\sum q_1 p_0}{\sum q_0 p_0}$$

GDP at comparable prices is equal to base GDP in base prices multiplied by the GDP quantity index:

$$\sum q_1 p_1 : I_{p1/0} = \sum q_1 p_0$$

GDP change due to chance of prices and quantity

Absolute change of GDP in current prices (pq)

$$\Delta_{GDP} = \Delta_{GDP}^p + \Delta_{GDP}^q$$

$$\Delta_{GDP} = GDP_1 - GDP_0$$

Change of prices (p)	Change of quality(q)
$\Delta_{GDP}^p = GDP_0 \cdot (I_{pq\text{ GDP}1/0} - I_{q\text{ GDP}1/0})$	$\Delta_{GDP}^q = GDP_0 \cdot (I_{q\text{ GDP}} - 1)$

GDP per capita

$$GDP_{per\ capita} = \frac{GDP}{P}$$

Gross Regional Product (GRP) per capita

$$GRP_{per\ capita} = \frac{GRP}{\bar{P}}$$

Index of GRP per capita

$$I_{GRP\text{ per capita}1/0} = \frac{GRP\text{ per capita}_1}{GRP\text{ per capita}_0} = \\ = \frac{\sum GRP\text{ per capita}_1 \cdot \bar{P}_1}{\sum \bar{P}_1} : \frac{\sum GRP\text{ per capita}_0 \cdot \bar{P}_0}{\sum \bar{P}_0}$$

Absolute change of GRP per capita:

$$\Delta_{GRP\text{ per capita}} = \frac{\sum GRP\text{ per capita}_1 \cdot \bar{P}_1}{\sum \bar{P}_1} - \frac{\sum GRP\text{ per capita}_0 \cdot \bar{P}_0}{\sum \bar{P}_0}$$

TESTS

Test 13-1 (select a correct answer)

Specify an indicator that is not included in GDP when you define it using production approach:

1. Intermediate consumption
2. Output
3. Taxes on production and imports
4. Taxes on products

Test 13-2 (select correct answers)

Specify indicators that are included in GDP when you define it using income approach:

1. Intermediate consumption
2. Compensation of employees
3. Output
4. Taxes on production and imports
5. Subsidies on production and imports
6. Taxes on products
7. Subsidies on products
8. Gross operating surplus and gross mixed income

Test 13-3 (select a correct answer)

Determine the missing indicator:

GDP	=	?	+	Gross capital formation	+	Balance of export and import	+	Statistical discrepancy
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Gross operating surplus and gross mixed income

1. Final consumption expenditures
2. Compensation of employees
3. Intermediate consumption

Test 13-4 (select a correct answer)

Determine the missing indicator:

GDP	=	Compensation of employees	+	Taxes on production and imports	-	Subsidies on production and imports	+	?
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1. Gross operating surplus and gross mixed income
2. Final consumption expenditures
3. Output
4. Intermediate consumption

Test 13-5 (Match the indicator in the left-hand column with its characteristics in the right-hand column):

1	Output	a	Remuneration in cash or in kind, paid by employer to employees for work done during reporting period
2	Intermediate consumption	b	Part of gross domestic product, which remains with producers after deducting expenses, related to compensation of employees and net taxes on production and imports. It measures profit (or loss), resulting from production, before accounting of payments or receiving of income from property
3	Final consumption expenditures	c	Includes: -gross fixed capital formation -changes in inventories -acquisitions less disposals of non-produced non-financial assets
4	Compensation of employees	d	Indicator, which characterizes error in construction of all accounts. Statistical discrepancy, which does not exceed 4-5% of GDP, is evidence of satisfactory quality of calculation
5	Statistical discrepancy	e	Consists of final consumption expenditures of: -households -government -non-profit institutions serving households
6	Gross operating surplus and gross mixed income	f	Total value of goods and services, resulting from production activity of resident units of economy
7	Gross capital formation	g	Consists of value of goods and services, transformed in production process during a reporting period

Test 13-6 (select correct answers)

Gross capital formation includes:

- 1) gross fixed capital formation
- 2) gross operating surplus and gross mixed income
- 3) changes in inventories
- 4) acquisitions less disposals of non-produced non-financial assets

Test 13-7 (select a correct answer)

Given the indicators of the system of national accounts:

Indicator	Value, trillion rubles
Output in basic prices	167
Intermediate consumption	84
Taxes on products	9
Other taxes on production	1,2
Subsidies on products	0,4
Other subsidies on production	0,3

Determine GDP using the production approach.

1. 91,6
2. 92,5
3. 83,9
4. 91,3

Test 13-8 (select a correct answer)

Taxes on production and imports includes:

- 1) only taxes on products
- 2) only other taxes on production
- 3) taxes on products and other taxes on production
- 4) subsidies on products and other subsidies on production

Test 13-9 (select a correct answer)

GDP in comparable prices is equal:

1. $\sum q_1 p_1 \cdot I_{q1/0} = \sum q_1 p_0$
2. $\sum q_0 p_0 \cdot I_{q1/0} = \sum q_1 p_0$
3. $\sum q_1 p_0 \cdot I_{q1/0} = \sum q_1 p_0$
4. $\sum q_0 p_1 \cdot I_{q1/0} = \sum q_1 p_0$

Test 13-10 (select a correct answer)

Adue to change of prices calculated as:

1. $GDP_0 \cdot (I_{qGDP} - 1)$
2. $GDP_0 \cdot (I_{pGDP} - 1)$
3. $\Delta_{GDP}^p = GDP_0 \cdot (I_{pq GDP1/0} - I_{q GDP1/0})$
4. $\Delta_{GDP}^p = GDP_0 \cdot (I_{p GDP1/0} - I_{q GDP1/0})$

PRACTICE

13.1. The following data are available for 2015 for the Russian Federation:

№	Indicator	Value, billion rubles
1	Output at basic prices	144740
2	Intermediate consumption	69977
3	Taxes on products	8739
4	Subsidies on products	270
5	Compensation of employees	38091
6	Taxes on production and imports	9787
7	Subsidies on production and imports	515
8	Gross operating surplus and gross mixed income	35869
9	Final consumption expenditures	
	including:	
	– households	43243
	– government	14544
	– non-profit institutions serving households	308
10	Gross capital formation	
	including:	
	– gross fixed capital formation	17265
	– changes in inventories	1355
	– acquisitions less disposals of valuables	2
11	Exports of goods and services	23860
12	Imports of goods and services	17149
14	Statistical discrepancy	-196

Source: Russian statistical yearbook. 2017

Determine GDP in 2015 using the methods:

1. Production approach.
2. Income approach.
3. Expenditure approach.

13.2. The following data are available (at current prices, billion rubles):

Output at basic prices	152324,9
Intermediate consumption	74816,6
Net taxes on products	8535,3
Net other taxes on production	878,0
Compensation of employees	40779,3
Final consumption expenditures	59822,7
Gross capital formation	20132,0
Exports of goods and services	22124,4
Imports of goods and services	17685,8
GDP deflator, percent of previous year	103,6

Perform the following tasks:

1) Determine:

- a) GDP at current prices using three methods;
- b) GDP structure;

2) Analyze the dynamics of the volume of GDP at current prices and at constant prices compared to the previous year, if the volume of GDP produced in market prices in the previous year amounted to 83232,6 billion rubles.

13.3. The following data are available (at current prices, billion rubles):

Output at basic prices	115792
Intermediate consumption	55980
Taxes on products	6991
Subsidies on products	216
Other taxes on production	839
Other subsidies on production	195
Compensation of employees	30473
Compensation of employees (including net wages received abroad and paid to non-residents in the country)	30227
Property income received from «the rest of the world»	1657
Property income transferred to «the rest of the world»	3146
Final consumption expenditure	46476
Gross capital formation	14898
Exports of goods and services	19088
Imports of goods and services	13719

Perform the following tasks:

- 1) Based on the above information fill in the tables:
 - a) "Production account";
 - b) "Generation of income account";
 - c) "Allocation of primary income account";
 - d) "Good and services account".
- 2) Determine GDP using the methods:
 - a) production approach;
 - b) income approach;
 - c) expenditure approach;
- 3) Determine the structure of GDP:
 - a) by sources of income;
 - b) by areas of use.

13.4. The following data are available (at current prices, billion rubles):

Output at basic prices	86844
Intermediate consumption	41985
Net taxes on products	5081
Net other taxes on production	483
Compensation of employees	22855
Net wages received abroad and paid to non-residents in the country	-85
Property income received from «the rest of the world»	1250
Property income transferred to «the rest of the world»	2340
Final consumption expenditure	
including:	
– households	25949
– general government	8726
– non-profit institutions serving households	185
Gross capital formation	
including:	
– gross fixed capital formation	10357
– changes in inventories	810
– acquisitions less disposals of valuables	3
Exports of goods and services	14200
Imports of goods and services	10150

Perform the following tasks:

- 1) Based on the above information fill in the tables:
 - a) "Production account";
 - b) "Generation of income account";
 - c) "Allocation of primary income account";
 - d) "Good and services account".
- 2) Determine GDP using the methods:
 - a) production approach;
 - b) income approach;
 - c) expenditure approach;
- 3) Determine the structure of GDP:
 - a) by sources of income;
 - b) by areas of use.

13.5. The indicators of the system of national accounts of Russia for 2017 are given:

Indicator	Value, billion rubles
Output at basic prices	166 754
Intermediate consumption	83 695
Taxes on products	9 265
Other taxes on production	1 243
Subsidies on products	286
Other subsidies on production	267

Source: Russian statistical yearbook. 2017

In 2016 GDP at current prices amounted to 86 149 billion rubles. GDP deflator index in 2017 compared to 2016 is equal to 105,3%.

Determine:

1. GDP at current prices in 2017.
2. Absolute change of GDP at current prices:
 - a) total;
 - b) due to change of prices;
 - c) due to change of quantity.

13.6. The following data on GDP dynamics are given:

Indicator	2014	2015	2016
GDP at current prices, billion rubles	79200	83232	86044
Quantity GDP index, % to the previous year	–	97,2	99,8

Source: Russian statistical yearbook. 2017

Determine:

1. GDP in constant prices (at prices of 2014 year) using extrapolation method.
2. Absolute growth and growth coefficient of GDP at current prices and at constant prices in 2016 compared to 2014 year.
3. GDP deflator indices in 2016 compared to 2015 and in 2015 compared to 2014.
4. Absolute change of GDP at current prices: in 2016 compared to 2015:
 - a) total;
 - b) due to change of prices;
 - c) due to change of quantity.

13.7. The following data on GDP dynamics are given:

Indicator	2014	2015	2016
GDP at current prices, billion rubles	79200	83232	86044
Average population, million persons	143,7	146,3	146,5
Quantity GDP index, % to the previous year	-	97,2	99,8

Source: Russian statistical yearbook. 2017

Determine:

1. GDP at constant prices (at prices of 2014 year) using extrapolation method.
2. GDP per capita at current prices and in constant prices (in prices 2014).
3. Growth coefficient of GDP per capita at current and at constant prices in 2016 compared to 2014.

13.8. Given the main indicators of the Generation of Income Account:

Indicator	2013	2014	2015
Compensation of employees	33792	37387	38091
Taxes on production and imports			
including:			
– taxes on products	9511	10551	8738
– other taxes on production	901	947	1049
Subsidies on production and imports			
including:			

Окончание

Indicator	2013	2014	2015
– subsidies on products	246	259	270
– other subsidies on production	103	235	245
Gross operating surplus and gross mixed income	29279	30808	35868
GDP deflator index, % to the previous year	–	107,5	108,2

Source: Russian statistical yearbook. 2017

Determine:

1. GDP at current prices for each year.
2. GDP at constant prices (at prices of 2013 year) using deflation method.
3. Absolute growth and growth coefficient of GDP at current prices and at constant prices in 2015 compared to 2013 year.

13.9. Data are available on the use of the gross domestic product in 2017 at current prices, billion rubles:

Indicator	2017 year			
	I quarter	II quarter	III quarter	IV quarter
1. Final consumption expenditures	15 291	15 772	16 568	17 132
2. Gross capital formation	3 256	5 401	6 453	6 851
3. Balance of export and import	1 752	1 008	653	1 512
4. Statistical discrepancy	250	-146	275	8

Source: Federal State Statistics Service <http://www.gks.ru>

Determine:

1. GDP at current prices for each quarter using expenditure approach.
2. Average quarter rate of increase.

13.10. Data on gross value added for two economic activities are available:

Economic activity	Gross Value Added, billion rubles		Deflator index in 2018 to 2017 г., %
	2017	2018	
Crude oil and gas production	4940	5349	107,4
Extraction of coal	281	262	92,9

Determine:

1. Change of gross value added on two economic activities (%).
2. Deflator index on average on two economic activities.

13.11. The following quarterly data on GDP are given

Quarter	GDP, trillion rubles		
	2015	2016	2017
I quarter	18,6	19,1	20,5
II quarter	19,9	20,6	22,0
III quarter	22,0	22,5	23,9
IV quarter	22,9	23,9	25,5
Total for a year:	83,4	86,1	91,9

Source: Federal State Statistics Service <http://www.gks.ru>

Define seasonal indices for each quarter.

13.12. Given the data on imports of goods and services:

	Years					
	2011	2012	2013	2014	2015	2016
Imports of goods and services, trillion rubles	12,0	13,8	14,9	16,4	17,2	17,7

Make a forecast of imports of goods and services for 2017:

- 1) using the average absolute growth;
- 2) using the trend line equation.

13.13. Regional indicators of the system of national accounts are available:

Federal district	Gross regional product, billion rubles		Average population, million persons	
	2011	2015	2011	2015
Central	16062	22714	38,5	39,0
North-Western	4785	6790	13,6	13,9
Southern	2777	4591	13,9	16,3

Determine:

1. Gross Regional Product (GRP) per capita for every federal district.
2. Relative and absolute change of GRP per capita for all federal districts.

13.14. Given the data on system of national accounts:

Table 1. System of National Accounts 2007 – 2014, bln. rub.

Indicator	2010	2011	2012	2013	2014
Output	82 055	100 960	113 092	123 165	133 110
Intermediate consumption	42 015	48 876	54 096	59 296	64 203
Taxes on products	6 463	8 413	9 412	9 511	10 551
Subsidies on products	194	215	244	246	259
Other taxes on production	2 032	694	793	901	947
Other subsidies on production	81	145	130	103	235
Compensation of employees	22 996	26 387	30 201	33 792	37 387
Gross operating surplus and gross mixed income	15 093	25 148	28 132	29 279	30 808
Final consumption expenditures of households	23 618	29 939	34 492	38 465	42 016
Final consumption expenditures of government	8 671	10 527	12 156	13 552	14 207
Final consumption expenditures of non-profit institutions serving households	226	225	247	257	288
Gross fixed capital formation	10 014	12 953	14 689	15 926	16 828
Changes in inventories	458	1 783	2 041	990	787
Acquisitions less disposals of valuables	0	-1	13	5	10
Exports of goods and services	13 529	16 865	18 325	18 863	21 426
Imports of goods and services	9 790	12 011	13 787	14 920	16 352
Statistical discrepancy	-417	2	-12	-4	-11

Table 2 – Quantity GDP index, % to the previous year

	2010	2011	2012	2013	2014
Quantity GDP index, % to the previous year	104,5	104,3	103,4	101,3	100,6

Perform the following tasks:

1. Determine GDP using the methods:
 - production approach.
 - income approach.
 - expenditure approach.
2. Calculate absolute change of GDP in each year compared to the previous year:
 - a) total;
 - b) due to change of prices;
 - c) due to change of quantity.

13.15. The following data on GDP dynamics are given:

Indicator	2014	2015
GDP at current prices, billion rubles	79199,7	83232,6
Quantity GDP index, % to 2014	100	97,2
The index of aggregate labour input, % to 2014	100	99,4

Determine in 2015 compared to 2014:

1. GDP deflator indices;
2. Labor productivity index (at constant prices);
3. Changes in GDP at current prices in absolute terms (billion rubles) and relative terms (percentage) due to each factor (price growth and changes in physical volume of production);
4. GDP growth at constant prices in absolute terms (billion rubles) and relative terms (percentage) due to each factor (changes in total labor costs and labor productivity).

13.16. The following data on GDP dynamics for 2011-2016 are given (as a percentage of the previous year):

Indicator	Years				
	2012	2013	2014	2015	2016
GDP at constant prices	103,7	101,8	100,7	97,2	99,8
GDP deflator	109,1	105,4	107,5	108,2	103,6

Determine:

1. Changes in the physical volume of GDP in 2016 compared to 2014;
2. For GDP the average annual growth rate and rate of increase growth at actual and constant prices for the period;
3. Deflator indices for 2012-2016 with constant base, taking as a base 2011 (2011=100%).

13.17. The following data on GDP dynamics are given:

Indicator	Years					
	2011	2012	2013	2014	2015	2016
GDP at current prices, trillion rubles	60,3	68,2	73,1	79,2	83,2	86,0
Gross Domestic Product (at constant prices), percent of previous year	104,3	103,7	101,8	100,7	97,2	99,8

Determine:

- 1) Absolute GDP growth at current prices (chain, basic, average).
- 2) GDP volume indices at current prices (chain, basic, average).
- 3) GDP deflator indices.
- 4) Average annual GDP growth at constant prices.
- 5) Absolute and relative GDP growth at current prices as a result of price changes.

13.18. The following data are available (at current prices, billion rubles):

Indicator	Years	
	2016	2018
Output at basic prices	113092,2	133110,2
Taxes on products	9411,8	10550,8
Other taxes on production	792,7	947,1
Subsidies on products	244,1	258,7
Other subsidies on production	129,7	235,0
Intermediate consumption	54095,9	64202,7
Final consumption expenditure	46895,8	56510,7
Gross fixed capital formation (including acquisition less disposals of valuables)	14689,2	16828,1
Changes in inventories	2041,0	786,5
Exports of goods and services	18324,8	21425,9
Imports of goods and services	13786,9	16351,5
Property income received from «the rest of the world»	1363,5	1623,0
Property income transferred to «the rest of the world»	3100,5	3815,3
Current transfers received from «the rest of the world»	511,5	671,6

Окончание

Indicator	Years	
	2016	2018
Current transfers transferred to «the rest of the world»	703,8	986,5
Compensation of employees (including net wages received abroad and paid to non-residents in the country)	29832,6	37006,0
Net wages received abroad and paid to non-residents in the country	-368,6	-381,1
Gross operating surplus and gross mixed income	28132,1	30808,3

Perform the following tasks:

1. Determine for each year:

- 1) GDP at current prices (using production approach);
- 2) GDP at current prices (using income approach);
- 3) GDP at current prices (using expenditure approach);
- 4) Statistical discrepancy;
- 5) Gross National Income (GNI);
- 6) The structure of GDP in areas of income use.

2. Analyze the structural changes in the composition of GDP.

13.19. The following data are available (at current prices, billion rubles):

Indicator	Value
Gross National Income (GNI)	67012
Compensation of employees (including net wages received abroad and paid to non-residents in the country)	32509
Net wages received abroad and paid to non-residents in the country	-114
Property income received from «the rest of the world»	1982
Property income transferred to «the rest of the world»	3691
Taxes on production and imports	7950
including:	
- taxes on products	7055
Subsidies on production and imports	419
including:	
-subsidies on products	227
Exports of goods and services	17699

Окончание

Indicator	Value
Imports of goods and services	14149
Intermediate consumption	59853
Final consumption expenditure	47858
Gross capital formation	16106

Determine:

1. Gross operating surplus and gross mixed income.
2. GDP at current prices (using income approach).
3. Output at basic prices.
4. Statistical discrepancy.
5. the structure of GDP by areas of use.



Statistics of fixed and current assets

Learning objectives:

- Fixed assets
- Total book value and residual book value
- Balance sheet of fixed assets
- Average value of fixed value
- Indicators of fixed assets movement, suitability and depreciation, and efficiency
- Types of current assets
- Average value of current assets
- Indicators of current assets efficiency

THEORY

Fixed assets (fixed capital) are produced assets that are to be used repeatedly or continuously for a long period of time, but not less than one year, for production of goods, rendering of market and non-market services, for management needs or for presentation to other organizations for temporary paid possession and use or for temporary use.

Types of fixed assets

1. Buildings
2. Structures
3. Machines and equipment

4. Vehicles
5. Other types of fixed assets

Data on availability of fixed assets are given at **total book value**. It is equal to the sum of net book value of fixed assets and accumulated depreciation values accounted in balance sheets of organizations. This value reflects the availability of fixed assets, excluding gradual loss of their consumer properties during operation.

Residual book value of fixed assets, indicated in balance sheets of organizations, reflects gradual loss of their consumer properties in amount of accumulated depreciation.

Balance sheets of fixed assets at total book value and residual book value

Balance sheets of fixed assets at total book value

Fixed assets at the beginning of the period	Received fixed assets for the period		Disposed fixed assets for the period		Fixed assets at the end of the period
	Total	including new fixed assets	Total	including liquidated fixed assets	
FA_{begin}^{total}	$FA_{recieved}^{total}$	$FA_{new}^{received}$	$FA_{disposed}^{total}$	$FA_{liquidated}^{total}$	FA_{end}^{total}

Total book value at the end of the period:

$$FA_{end}^{total} = FA_{begin}^{total} + FA_{recieved}^{total} - FA_{disposed}^{total}$$

Balance sheets of fixed assets at residual book value

Fixed assets at the beginning of the period	Received fixed assets for the period		Disposed of fixed assets for the period		Depreciation for the period	Fixed assets at the end of the period
	Total	including new fixed assets	Total	including liquidated fixed assets		
$FA_{begin}^{residual}$	$FA_{recieved}^{total}$	$FA_{new}^{received}$	$FA_{disposed}^{residual}$	$FA_{liquidated}^{residual}$	D	$FA_{end}^{residual}$

Residual book value at the end of the period:

$$FA_{end}^{residual} = FA_{begin}^{residual} + FA_{recieved}^{total} - FA_{disposed}^{residual} - D$$

Average value of fixed assets

Data	Formula
Data on fixed assets at the beginning and end of the period (Simple arithmetic mean)	$\overline{FA} = \frac{FA_{begin} + FA_{end}}{2}$
Data on fixed assets for equal intervals (Chronological mean)	$\overline{FA} = \frac{\frac{1}{2}FA_1 + FA_2 + \dots + \frac{1}{2}FA_n}{n - 1}$
Data on fixed assets for unequal periods of time (Weighted arithmetic mean)	$\overline{FA} = \frac{\sum FA_j \cdot t_j}{\sum t_j}$
Balance method	$\overline{FA} = FA_{begin} + \frac{FA_{received}^{total}}{12} k - \frac{FA_{disposal}^{total}}{12} (12 - k)$

Indicators of fixed assets movement

Indicator	Russian equivalent	Formula
Receipt ratio	Коэффициент поступления	$K_{receipt} = \frac{FA_{received}}{FA_{end}^{total}} \cdot 100\%$
Renovation ratio	Коэффициент обновления	$K_{renovation} = \frac{FA_{received\ new}}{FA_{end}^{total}} \cdot 100\%$
Disposal ratio	Коэффициент выбытия	$K_{disposal} = \frac{FA_{disposed}^{total}}{FA_{begin}^{total}} \cdot 100\%$
Liquidation ratio	Коэффициент ликвидации	$K_{liquidation} = \frac{FA_{liquidated}^{total}}{FA_{begin}^{total}} \cdot 100\%$
Replacement ratio	Коэффициент замены	$K_{replacement} = \frac{FA_{liquidated}^{total}}{FA_{received\ new}} \cdot 100\%$

Indicators of fixed assets suitability and depreciation

Indicator	Russian equivalent	Formula
Depreciation - at the beginning of the period - at the end of the period	Износ на начало периода на конец периода	$D_{begin} = FA_{begin}^{total} - FA_{begin}^{residual}$ $D_{end} = FA_{end}^{total} - FA_{end}^{residual}$

Окончание

Indicator	Russian equivalent	Formula
Depreciation ratio - at the beginning of the period - at the end of the period	Коэффициент износа	$K_{depreciation_{begin}} = \frac{D_{begin}}{FA_{begin}^{total}} \cdot 100\%$ $K_{depreciation_{end}} = \frac{D_{end}}{FA_{end}^{total}} \cdot 100\%$
Suitability ratio - at the beginning of the period - at the end of the period	Коэффициент годности	$K_{suitability_{begin}} = \frac{FA_{begin}^{residual}}{FA_{begin}^{total}} \cdot 100\%$ $K_{suitability_{end}} = \frac{FA_{end}^{residual}}{FA_{end}^{total}} \cdot 100\%$
Relationship between depreciation ratio and suitability ratio		$K_{depreciation_{begin}} + K_{suitability_{begin}} = 100\%$ $K_{depreciation_{end}} + K_{suitability_{end}} = 100\%$

Indicators of fixed assets efficiency

Indicator	Russian equivalent	Formula
Efficiency ratio	Фондоотдача	$E = \frac{Production\ volume(PV)}{\overline{FA}}$
Output ratio	Фондоёмкость	$O = \frac{\overline{FA}}{Production\ volume} = \frac{1}{E}$
Capital-labour ratio	Фондовооруженность	$CL = \frac{\overline{FA}}{Average\ headcount}$

Factor analysis

$$PV = E \cdot \overline{FA}$$

Absolute change of production volume (PV)

$$\Delta_{PV} = PV_1 - PV_0 = \Delta_{PV}^E + \Delta_{PV}^{\overline{FA}}$$

Change of efficiency ratio (E)	Change of average value of fixed assets (\overline{FA})
$\Delta_{PV}^E = (E_1 - E_0) \cdot \overline{FA}_1$	$\Delta_{PV}^{\overline{FA}} = E_0 \cdot (\overline{FA}_1 - \overline{FA}_0)$

Types of current assets

1. Raw materials, intermediate goods
2. Goods in process
3. Future expenses
4. Finishes output
5. Goods for resale
6. Cash
7. Financial investments
8. Other

Average value of current assets

Data	Formula
Data on current assets at the beginning and end of the period (Simple arithmetic mean)	$\overline{CA} = \frac{CA_{begin} + CA_{end}}{2}$
Data on current assets for equal intervals (Chronological mean)	$\overline{CA} = \frac{\frac{1}{2}CA_1 + CA_2 + \dots + \frac{1}{2}CA_n}{n - 1}$
Data on current assets for unequal periods of time (Weighted arithmetic mean)	$\overline{CA} = \frac{\sum CA_i \cdot t_i}{\sum t_i}$

Indicators of current assets efficiency

Indicator	Russian equivalent	Formula (or designation)
Sales proceeds	Выручка от реализации	SP
Average value of current assets	Средняя стоимость оборотных активов	\overline{CA}
Turnover ratio	Коэффициент оборачиваемости	$K_{turnover} = \frac{SP}{\overline{CA}} \text{ (turns)}$
Consolidation ratio	Коэффициент закрепления	$K_{consolidation} = \frac{\overline{CA}}{SP}$ $K_{consolidation} = \frac{1}{K_{turnover}}$
Average duration of one turn	Средняя продолжительность одного оборота	$\bar{t} = \frac{T}{K_{turnover}} \text{ (turns)}$ <p>– duration of the period in days month – 30 days quarter – 90 days half year – 180 days year – 360 days</p>

Factor analysis

$$\overline{CA} = K_{consolidation} \cdot SP$$

Absolute change of average value of current assets (\overline{CA})

$$\Delta \overline{CA} = \overline{CA}_1 - \overline{CA}_0 = \Delta_{CA}^K + \Delta_{CA}^{SP}$$

<p>Change of consolidation rate (K_{cons})</p> $\Delta_{CA}^K = (K_{cons1} - K_{cons0}) \cdot SP_1$	<p>Change of sales proceeds (SP)</p> $\Delta_{CA}^{SP} = K_{cons0} \cdot (SP_1 - SP_0)$
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TESTS

Test 14-1 (Match the nature of the data in the left-hand column with the formula in the right-hand column):

1	Data on fixed assets at the beginning and end of the perio	a	$\overline{FA} = FA_{begin} + \frac{FA_{recieved}^{total}}{12} k - \frac{FA_{disposal}^{total}}{12} (12 - k)$
2	Data on fixed assets for unequal periods of time	b	$\overline{FA} = \frac{\sum FA_i \cdot t_i}{\sum t_i}$
3	Balance method	c	$\overline{FA} = \frac{\frac{1}{2}FA_1 + FA_2 + \dots + \frac{1}{2}FA_n}{n - 1}$
4	Data on fixed assets for equal intervals	d	$\overline{FA} = \frac{FA_{begin} + FA_{end}}{2}$

Test 14-2 (select a correct answer)

Total book value at the end of the period is calculated as:

1. $FA_{begin}^{total} + FA_{recieved}^{total} - FA_{disposed}^{total}$
2. $FA_{begin}^{total} + FA_{recieved}^{total} + FA_{disposed}^{total}$
3. $FA_{begin}^{residual} + FA_{recieved}^{total} - FA_{disposed}^{residual} + D$
4. $FA_{begin}^{residual} + FA_{recieved}^{total} + FA_{disposed}^{residual} + D$
5. $FA_{begin}^{residual} + FA_{recieved}^{total} - FA_{disposed}^{residual} - D$

Test 14-3 (select a correct answer)

Residual book value at the end of the period is calculated as:

1. $FA_{begin}^{total} + FA_{recieved}^{total} - FA_{disposed}^{total}$
2. $FA_{begin}^{total} + FA_{recieved}^{total} + FA_{disposed}^{total}$
3. $FA_{begin}^{residual} + FA_{recieved}^{total} - FA_{disposed}^{residual} + D$
4. $FA_{begin}^{residual} + FA_{recieved}^{total} + FA_{disposed}^{residual} + D$
5. $FA_{begin}^{residual} + FA_{recieved}^{total} - FA_{disposed}^{residual} - D$

Test 14-4 (Match the indicator in the left-hand column with the formula in the right-hand column):

1	Receipt ratio	a	$\frac{FA_{received\ new}}{FA_{end}^{total}} \cdot 100\%$
2	Disposal ratio	b	$\frac{FA_{liquidated}^{total}}{FA_{begin}^{total}} \cdot 100\%$
3	Replacement ratio	c	$\frac{FA_{recieved}}{FA_{end}^{total}} \cdot 100\%$
4	Renovation ratio	d	$\frac{FA_{liquidated}^{total}}{FA_{received\ new}} \cdot 100\%$
5	Liquidation ratio	e	$\frac{FA_{disposed}^{total}}{FA_{begin}^{total}} \cdot 100\%$

Test 14-5 (select a correct answer)

Efficiency ratio is calculated as:

1. $\frac{\overline{FA}}{\text{Production volume}}$
2. $\frac{\overline{FA}}{\text{Average headcount}}$
3. $\frac{\text{Production volume}}{\overline{FA}}$
4. $\frac{\text{Average headcount}}{\overline{FA}}$

Test 14-6 (select a correct answer)

If you have data on current assets for equal intervals you use:

1. Simple arithmetic mean
2. Weighted arithmetic mean
3. Chronological mean
4. Geometric mean

Test 14-7 (select a correct answer)

Given the following data (thou. rub):

- fixed assets at the beginning of the period – 500;
- fixed assets at the end of the period – 475;
- received fixed assets – 25;
- disposed fixed assets – 50.

Receipt ratio is equal:

- | | |
|---------|-----------|
| 1. 5,0% | 3. 10,0% |
| 2. 5,3% | 4. 10,5%. |

Test 14-8 (select a correct answer)

Given the following data (thou. rub):

- fixed assets at the beginning of the period – 500;
- fixed assets at the end of the period – 475;
- received fixed assets – 25;
- disposed fixed assets – 50.

Disposal ratio is equal:

- | | |
|---------|-----------|
| 1. 5,0% | 3. 10,0% |
| 2. 5,3% | 4. 10,5%. |

Test 14-9 (select correct answers)

Indicators of fixed assets movement are:

1. Liquidation ratio
2. Replacement ratio
3. Depreciation ratio
4. Receipt ratio
5. Efficiency ratio
6. Suitability ratio
7. Renovation ratio
8. Output ratio
9. Disposal ratio

Test 14-10 (select correct answers)

Indicators of fixed assets efficiency are:

1. Liquidation ratio
2. Replacement ratio
3. Depreciation ratio
4. Receipt ratio
5. Efficiency ratio
6. Suitability ratio
7. Renovation ratio
8. Output ratio
9. Disposal ratio

PRACTICE

14.1. There are data on fixed assets in the first half of 2018:

Data	Value of fixed assets, million rubles
01.01	380,7
01.02	390,4
01.03	390,7
01.04	328,8
01.05	390,2
01.06	405,3
01.07	400,0

Determine average value of fixed assets for: a) the I quarter; b) the II quarter; c) the first half of the 2018 year.

14.2. Given the data of balance sheets of fixed assets at total book value and residual book value:

Balance sheets of fixed assets at total book value, thousand rubles

Fixed assets at the beginning of the period	Received fixed assets for the period		Disposal of fixed assets for the period		Fixed assets at the end of the period
	Total	including new fixed assets	Total	including liquidated fixed assets	
10500	3200	2200	1800	500	

Balance sheets of fixed assets at residual book value, thousand rubles

Fixed assets at the beginning of the period	Received fixed assets for the period		Disposal of fixed assets for the period		Depreciation for the period	Fixed assets at the end of the period
	Total	including new fixed assets	Total	including liquidated fixed assets		
7200	3200	2200	700	200	1200	

Determine:

1. Total book value and residual book value at the end of the year.
2. Indicators of fixed assets movement.
3. Indicators of fixed assets suitability and depreciation at the beginning and at the end of the year.

14.3. Given the data for 2018 year:

Indicator	Value
Total book value of fixed assets at the beginning of the year, thou. rubles	15 500
Depreciation rate at the beginning of the year, %	38
Received new fixed year for a year, thou. rubles	3 300
Total value of disposed fixed assets, thou. rubles	2 800
Residual value of disposed fixed assets, thou. rubles	1 960
Depreciation for the year, thou. rubles	2 362
Production volume, thou. rubles	56 700

Perform the following tasks:

1. Construct the balance of fixed assets:
 - a) at total book value;
 - b) at residual book value.
2. Determine the indicators of fixed assets movement.
3. Determine the indicators of suitability and depreciation.
4. Determine the indicators of fixed assets efficiency.

14.4. Given the data for 2018 year:

At the beginning of the year:

- total book value of fixed assets – 24 300 thou. rubles;
- suitability ratio – 82 %.

In April were sold fixed assets, the total book value of which amounted to 3 480 thousand rubles; their depreciation on that date – 2 110 thou. rubles.

In June new fixed assets were acquired in the amount of 8 650 thousand rubles.

In October fixed assets were sold in the amount of 4 670 thou. rubles, their residual book value – 2 300 thou. rubles.

Depreciation for the year – 3 402 thou. rubles.

Production volume for the year – 109 350 thou. rubles.

Determine:

1. Total book value at the end of the year.
2. Residual book value at the end of the year.
3. Average value of fixed assets.
4. Indicators of fixed assets movement.
5. Indicators of fixed assets suitability and depreciation at the beginning and at the end of the year.
6. The indicators of fixed assets efficiency.

14.5. Given the total value of fixed assets at the beginning of each month:

Month	Value, thousand rubles
January (current year)	5500
February	5740
March	5620
April	5650
May	5680
June	5800
July	5800
August	5875
September	6550
October	6550
November	6550
December	5900
January (next year)	5900

Production volume for the year – 21 220 thousand rubles.

Determine indicators of fixed assets efficiency.

14.6. Given the following data:

- residual value of fixed assets at the beginning of the year – 68 500 thou. rubles;
- depreciation at the beginning of the year – 6 500 thou. rubles;
- disposal ratio – 8%;
- the value of new received fixed assets – 4 500 thou. rubles.

Calculate the total value of fixed assets at the end of the year.

14.7. Given the following data:

- renovation ratio – 3%
- the value of new received fixed assets – 450 thou. rubles;
- depreciation at the end of the year – 2 200 thou. rubles.

Calculate the suitability ratio of fixed assets at the end of the year.

14.8. The following data are available for the chemical plant for two months:

Indicator	January	February
Production volume, thousand rubles	211140	265125
Average value of fixed assets, thousand rubles	5100	5250

Determine:

1. Fixed assets efficiency ratio and its absolute and relative change.
2. Absolute change of production volume due to:
 - a) change of average value of fixed assets;
 - b) change of fixed assets efficiency ratio.

14.9. Given the following data:

Efficiency ratio, mln. rub/ mln. rub	Number of branches	Average annual value of fixed assets per branch, mln. rub.	Capital-labor ratio, thou. rub / person
up to 4	5	30	100
4 – 6	8	44	130
6 and more	7	50	200

Determine for the whole organization:

1. Average efficiency ratio.
2. Average annual value of fixed assets per branch.
3. Average capital-labor ratio.

14.10. Given the total value of fixed assets at the beginning of each month in 2018:

Month	Value, thousand rubles
January (current year)	1200
February	1300
March	1300
April	1300
May	1600
June	1600
July	1400
August	1400
September	1900
October	1900
November	1700
December	1700

The total cost of fixed assets at the beginning of January 2019 increased by 300 thousand rubles. The volume of production in 2018 was 6 970 thousand rubles. The number of employees amounted to 200 people in 2018. In 2019, fixed assets efficiency ratio increased by 2.5%, while the average annual value of fixed assets increased by 1,058 times.

Determine:

1. The average annual value of fixed assets in 2018.
2. Fixed assets efficiency ratio, output ratio and capital-labor ratio in 2018.
3. Absolute and relative change of production volume in 2019 compared to 2018.
4. Absolute change of production volume in 2019 compared to 2018 due to:
 - a) change of average value of fixed assets;
 - b) change of fixed assets efficiency ratio.

14.11. There are the following data on the organization:

The average annual value of fixed assets in 2019 compared to 2018 increased by 15% and amounted to 3 255 thousand rubles. Fixed assets efficiency ratio for this period increased by 8,6% and amounted to 4,85 rubles.

Determine for this period in absolute and relative terms:

- 1) the change in output ratio;
- 2) the change in volume of production.

14.12. There are the following data on the organization:

The volume of production (at comparable prices) in 2019 compared to 2018 increased by 20% and amounted to 20 440 thousand rubles. The average annual value of fixed assets for this period increased by 15% and amounted to 4 542 thousand rubles.

Determine:

- 1) the change in fixed assets efficiency ratio;
- 2) absolute change of production volume in 2019 compared to 2018 due to factors.

14.13. The following data are available for the chemical plant for two months:

Indicator	March	April
Production volume, thousand rubles	20 500	20 700
Average value of fixed assets, thousand rubles	4 100	4 200
Number of staff, persons	100	120

Determine:

- 1) Fixed assets efficiency ratio and its absolute and relative change;
- 2) capital-labor ratio and its absolute and relative change;
- 3) indicator of labour productivity and its absolute and relative change;
- 4) the increase in the volume of production, including due to certain factors.

14.14. There are the following data on the organization:

Branch	The average annual value of fixed assets, thousand rubles		Production volume, thousand rubles	
	2018	2019	2018	2019
1	400	500	1 200	1 750
2	600	800	2 400	3 600

Determine the absolute and relative change of fixed assets efficiency ratio in 2019 compared to 2018 as a whole and due to each factor:

- a) changes in the structure of value of fixed assets;
- b) changes in fixed assets efficiency ratio in branches.

14.15. Given the data on current assets:

- 01.01 – 1212 thousand rubles;
- 01.04 – 1284 thousand rubles;
- 01.07 – 1303 thousand rubles.

Sales proceeds for the first half of the year – 13860 thou. rub.

Determine indicators of current assets efficiency.

14.16. Given the data for two years:

Indicator	2017	2018
Sales proceeds, thousand rubles	8100	8875
Average value of current assets, thousand rubles	785	823

Determine the change of turnover ratio (%).

14.17. Given the data for two months:

Indicator	January	March
Sales proceeds, thousand rubles	4200	4500
Average value of current assets, thousand rubles	1480	1500

Determine:

1. Indicators of current assets efficiency.
2. Absolute change of average value of current assets due to change of consolidation rate and sales proceeds.

14.18. Enterprise reporting data is the following:

- average value of current assets – 1810 thousand rubles;
- sales proceeds – 17900 thousand rubles

Calculate average duration of one turn and consolidation ratio.

14.19. According to the reporting data average duration of one turn decreased by 8,5% and average value of current assets decreased by 5,0 %.

Determine the change of sales proceeds (%).

14.20. Given the data for the second half of the year:

Data	Current assets, thousand rubles
01.07	3080
01.08	3140
01.09	3230
01.10	3260

Окончание

Data	Current assets, thousand rubles
01.11	3250
01.12	3380
01.01 (of the next year)	3400

Sales proceeds, thousand rubles:

1. III quarter – 12720;
2. IV quarter – 14940.

Determine the indicators of current assets efficiency for the III and the IV quarters.

14.21. Given the data for the second half of the year:

Indicator	July	August	September	October	November	December	January
Current assets at the beginning of the month, thou. rub.	640	634	647	656	664	660	662
Sale proceeds for the month, thou. rub.	1958	2010	2012	2029	2100	2084	2088

Determine:

1. Average value of current assets in the III and the IV quarters.
2. Indicators of current assets efficiency in the III and the IV quarters.
3. Absolute change of average value of current assets in the IV quarter comparing to the III quarter due to change of consolidation rate and sales proceeds.

14.22. Given the following data for the organization:

The average value of current assets in the reporting year decreased by 6,4% to 2 450 thousand rubles, while the increase in the average duration of one turnover from 120 to 125 days.

Determine:

1. Sales proceeds in the reporting and base year;
2. Absolute change of average value of current assets due to change of consolidation rate and sales proceeds.

14.23. Given the data for the II quarter of the year on the company:

Indicators	Branch		
	№1	№2	№3
Average value of current assets, thou. rub.	1 200	1 800	1 600
Average duration of one turn, days	10	9	6

Determine for the company as a whole:

1. The average turnover ratio of current assents;
2. The average duration of one turn.

CASES

Case 1. Absolute and relative values. Grouping of data

State Office of Public Prosecutor called “Top-10 the most criminal” regions of Russia <https://www.rbc.ru/rbcfreenews/59c8338b9a7947c8b5af4bfe>



Moscow, Moscow region, and St. Petersburg behind the top ten were the leaders of the rating. Is that really so?

The data of the State Office of Public Prosecutor is the following:

Region	Population, thousand persons	Crimes
Белгородская область	1 550	13 715
Брянская область	1 226	15 821
Владимирская область	1 397	19 138

Продолжение

Region	Population, thousand persons	Crimes
Воронежская область	2 334	34 370
Ивановская область	1 030	14 405
Калужская область	1 010	16 193
Костромская область	652	8 778
Курская область	1 120	12 190
Липецкая область	1 156	12 369
Московская область	7 319	88 297
Орловская область	760	9 313
Рязанская область	1 130	9 524
Смоленская область	959	13 677
Тамбовская область	1 050	12 178
Тверская область	1 305	20 992
Тульская область	1 506	12 008
Ярославская область	1 272	18 401
г. Москва	12 330	173 898
Республика Карелия	630	13 464
Республика Коми	857	19 045
Архангельская область	1 174	20 757
Вологодская область	1 188	25 724
Калининградская область	976	13 224
Ленинградская область	1 779	17 535
Мурманская область	762	15 061
Новгородская область	616	11 986
Псковская область	646	9 498
г. Санкт-Петербург	5 226	52 351
Республика Адыгея	452	4 639
Республика Калмыкия	279	2 951
Краснодарский край	5 514	71 509
Астраханская область	1 019	13 893

Продолжение

Region	Population, thousand persons	Crimes
Волгоградская область	2 546	38 742
Ростовская область	4 236	57 691
Республика Дагестан	3 016	15 810
Республика Ингушетия	473	1 790
Кабардино-Балкарская Республика	862	7 671
Карачаево-Черкесская Республика	468	3 519
Республика Северная Осетия-Алания	704	7 036
Чеченская Республика	1 394	3 723
Ставропольский край	2 802	34 336
Республика Башкортостан	4 071	65 343
Республика Марий Эл	686	7 214
Республика Мордовия	807	8 501
Республика Татарстан	3 869	48 150
Удмуртская Республика	1 517	26 219
Чувашская Республика	1 237	13 836
Пермский край	2 634	50 076
Кировская область	1 298	21 750
Нижегородская область	3 260	40 238
Оренбургская область	1 995	27 230
Пензенская область	1 349	12 319
Самарская область	3 206	44 102
Саратовская область	2 488	25 026
Ульяновская область	1 258	15 321
Курганская область	862	19 353
Свердловская область	4 330	63 454
Тюменская область	3 616	56 064
Челябинская область	3 501	64 576
Республика Алтай	215	4 832
Республика Бурятия	982	23 479

Окончание

Region	Population, thousand persons	Crimes
Республика Тыва	316	10 914
Республика Хакасия	537	11 052
Алтайский край	2 377	44 576
Забайкальский край	1 083	29 658
Красноярский край	2 867	57 248
Иркутская область	2 413	47 009
Кемеровская область	2 718	58 414
Новосибирская область	2 762	45 201
Омская область	1 979	29 352
Томская область	1 077	20 432
Республика Саха (Якутия)	960	12 375
Камчатский край	316	5 858
Приморский край	1 929	39 174
Хабаровский край	1 335	27 008
Амурская область	806	18 656
Магаданская область	146	2 986
Сахалинская область	487	12 248
Еврейская автономная область	166	3 424
Чукотский автономный округ	50	800

1. Range the regions by the number of crimes. List the regions with the highest number of crimes.

2. Define for each region number of crimes per 10 000 persons.

3. Range the regions on the number of crimes per 10 000 persons. List regions with the highest number of crimes per 10 000 persons.

4. Group regions by the number of crimes per 10 000 persons. The number of intervals define by yourself. For each group count the number of regions and the nu number of crimes per 10 000 persons. List regions for each group.

5. Did the State Office of Public Prosecutor correctly identify the most criminal” regions of Russia?

**Case 2. Grouping of data. Measures of central tendency.
Measures of variance. Shape of a distribution.
Sample method. Correlation and regression analysis**



Given the 30% simple random sample of banks:

No of bank	Credit exposure, million rubles	Profit, million rubles	No of bank	Credit exposure, million rubles	Profit, million rubles
1	162	49	16	180	63
2	43	6	17	140	50
3	194	72	18	185	69
4	95	29	19	160	50
5	184	67	20	162	58
6	183	65	21	194	72
7	76	17	22	214	74
8	121	22	23	216	76
9	184	70	24	228	86
10	101	17	25	205	72
11	147	75	26	221	77
12	130	38	27	243	91
13	146	58	28	248	94
14	187	71	29	259	97
15	173	60	30	248	92

The goal is to analyze the distribution of banks and determine the relationship between the main indicators.

Task 1

1. Group the data on X. The number of groups define by a Sturges formula.
2. Construct a histogram and frequency polygon.
3. Calculate measures of central tendency – mean, mode, median.
4. Define measures of position – quartiles (, ,), deciles (and) and decile ratio.
5. Calculate measures of variance – range, mean liner deviation, standard deviation, quartile deviation, coefficient of variation.
6. Define skewness.
7. Make a conclusion.

Task 2

Using the results of task 1 construct 95,4% confidence interval:

- 1) for the population mean ()
 - 2) for the population proportion of x-values more than sample mean.
- Make a conclusion.

Task 3

1. Check the possibility of using the correlation method:
 - define mean, standard deviation and coefficient of variation of x-value.
 2. Define existence and direction of relationship between X and Y using:
 - scatterplot;
 - method of analytical group.
 3. Estimate the strength of the relationship counting:
 - Karl Pearson's coefficient of correlation;
 - correlation ratio.
 4. Determine the equation of the least-squares regression line. Give interpretation of parameters a and b.
 5. Make a prediction of Y-value if $X=250$.
- Make a conclusion about the distribution of banks by credit exposure and relationship between credit exposure and profit.

Case 3. Population statistics



Given the data on demographic indicators from Federal State Statistics Service's website <http://www.gks.ru> (table 1, table 2, table 3, table 4).

The goal is to analyze demographic situation in Russia for the period from 2008 to 2013.

**Table 1. POPULATION DISTRIBUTION BY AGE GROUPS
(as of January 1, thousand persons)**

	2008	2009	2010	2011	2012	2013	2014
Total population	142748	142737	142857	142865	143056	143347	143667
Age groups, years:							
0-4	7433	7671	7968	8051	8380	8687	8899
5-9	6638	6783	7091	7117	7261	7441	7662
10-14	7056	6891	6610	6601	6567	6689	6823
15-19	10485	9650	8389	8237	7631	7152	6956
20-24	12457	12389	12169	12122	11599	10849	9971
25-29	11358	11667	11982	12012	12328	12556	12522
30-34	10537	10696	10980	11016	11116	11346	11661
35-39	9705	9885	10172	10211	10380	10459	10614
40-44	9800	9409	9241	9251	9340	9563	9751
45-49	11929	11634	10672	10561	10023	9545	9187
50-54	11037	11272	11483	11509	11560	11436	11184

Окончание

	2008	2009	2010	2011	2012	2013	2014
55-59	9501	9755	10022	10063	10215	10382	10634
60-64	5014	5916	7832	7982	8380	8690	8948
65-69	6687	5565	4002	3913	3896	4453	5269
70 and more	13111	13554	14210	14219	14380	14099	13587
Main demographic groups:							
Under working age	22842	22854	23126	23209	23568	24110	24717
Working age	89745	89342	87983	87847	87055	86137	85162
Over working age	30161	30541	31714	31809	32433	33100	33788

**Table 2. FERTILITY AND MORTALITY
(persons)**

Year	Persons	
	Births	Deaths
2000	1266800	2225332
2001	1311604	2254856
2002	1396967	2332272
2003	1477301	2365826
2004	1502477	2295402
2005	1457376	2303935
2006	1479637	2166703
2007	1610122	2080445
2008	1713947	2075954
2009	1761687	2010543
2010	1788948	2028516
2011	1796629	1925720
2012	1902084	1906335
2013	1895822	1871809
2014	1942683	1912347

**Table 3. MARRIAGES AND DIVORCES
(units)**

Year	Marriages	Divorces
2000	897327	627703
2001	1001589	763493
2002	1019762	853647
2003	1091778	798824
2004	979667	635835
2005	1066366	604942
2006	1113562	640837
2007	1262500	685910
2008	1179007	703412
2009	1199446	699430
2010	1215066	639321
2011	1316011	669376
2012	1213598	644101
2013	1225501	667971
2014	1225985	693730

Table 4. MIGRATION (persons)

	2008	2009	2010	2011	2012	2013
Arrivals to Russia – total	281614	279907	191656	356535	417681	482241
of them:						
from CIS countries	261170	261495	171940	310549	363955	422738
from other countries	20444	18412	19716	45986	53726	59503
Departures from Russia – total	39508	32458	33578	36774	122751	186382
of them:						
to CIS countries	25542	20326	21206	22568	95572	147853
to other countries	13966	12132	12372	14206	27179	38529

Using the data from tables 1, 2 and 3 fill in table 5 with demographic indicators for 2013:

Table 5. Demographic indicators

№	Indicator	Value
1	Population at the beginning of 2013 year, thousand persons	
2	Of the total population – population in age groups, thousand persons: – under working age – working age – over working age	
3	Population aged 60 years and older, thousand persons	
4	Births, persons	
5	Deaths, persons	
6	Marriages	
7	Divorces	
8	Population at the beginning of 2014 year, thousand persons	

1. Define the age structure of Russian population by main age groups (in %, accurate to 0.1)
(в % с точностью до 0,1).
2. Determine dependency ratio:
 - child dependency ratio;
 - old-aged dependency ratio;
 - total dependency ratio.
3. Calculate index of aging.
4. Define average annual population (mid-point population).
5. Calculate the following indicators:
 - crude birth rate;
 - crude death rate;
 - natural increase;
 - natural increase rate;
 - vitality rate;
 - marriage rate and divorce rate;
 - stability of marriages.

Using the data from table 4 fill in table 6 with migration indicators for 2013:

Table 6. Migration indicators

№	Indicator	Value
1	Arrivals to Russia, persons	
	– from CIS countries	
	– from other countries	
2	Departures from Russia, persons	
	– to CIS countries	
	– to other countries	
3	Average annual population	Take from Task 1

Calculate:

6. Migration turnover and net migration

7. Relative migration indicators

- arrival rate
- departure rate
- migration turnover rate
- net migration rate

Make a conclusion about demographic situation in Russia.

THE ANSWERS TO THE TASKS

Number of task	Answers
3.1	1) 131,5%; 2) 124,0%; 3) 94,3%
4.3	for male: 1) 4,8; 2) 4; 3) 4; 4) 3; 6
4.4	for department 1: 73,5; 76,25; 75
4.8	in April 55 457,14
5.9	11,92; 8,2%
5.12	1) 87,04; 2) 56,8; 3) 143,84; 4) 60,5%; 5) 0,78
5.20	1) 0,1; 0,04; 1,127; 2) 0,369; 3) 3,002; 4) 3,398; 5) 88,3%; 6) 0,94
6.1	1) 2,64; 4,36; 2) 5,8%; 14,2%
6.2	1) 42,41; 45,09; 2) 7,8%; 12,6%
6.7	1) 26,79; 2) 15,9; 3) 0,239; 4) 14,9%; 5) 25,9; 26,4; 6) 0,72; 7) 26,07; 27,51; 8) 0,042; 9) 81,4%; 89,8%
7.1	2) 0,85; 3) 3,8; 4,82; 4) 8,62
7.7	0,83; 0,39
7.10	0,6
8.2	1) 12203,88; 5176,25; 2) 121; 510
8.3	2) 448,2; 3) 12,75; 102,9%; 2,9%; 4,4
8.4	1) 4257; 2) 4168; 3) 3923; 4) 4045,4
8.7	a) 593,55; b) 761,43; c) 764,52; d) 704,67

Number of task	Answers
8.9	a) 84,36; b) 3,975; c) 104,8%; d) 4,8%; e) 20,6%
8.12	2.a) 2 654,2; d) 2,97%
8.21	for January 82,8%
9.1	2) 72,5%; 104,7%; 75,9%; 3) -1740; -2080; 340
9.5	1) 106,0%; 2) 105,7%
9.19	9,46; 0,01; 9,45
9.25	2) 101%; 99,5; 100,5; 3) 17,44; 36,19; -18,75
10.15	1) 10 058; 2) 9 887; 3) 14,5; 10,0; 4,5; 5,9; 4) 72,7
11.8	649,98; 65,9%
11.14	1) 207; 2) 203
11.15	1) 113; 2) 92; 3) 131
11.16	180; 169
11.17	2) 20,2; 3) 7,5
11.18	1. a) 2; 2,5; b) 15; 19; c) 315; 395; 4) a) -4000; b) 24000; 5) 0,5; 0,4; 7) a) 10000; b) -12000
12.15	100,9; 99,4; 97,0; 98,4; 95,7; 98,9
12.16	22 344; 8 500; 57 152
13.5	1) 92 038; 2) a) 5 889; b) 4 632,5; c) 1 256,5
13.19	1) 28 681; 2) 67 514; 3) 121 860; 4) 1 321
14.4	1) 24 800; 2) 21 504; 3) 25 526,7; 4) 34,9%; 33,5%; 5) 18%; 86,7%; 13,3%; 6) 4,28; 0,23
14.8	1) 41,4; 50,5; 9,1; 122%; 2) a) 6 210; b) 47 775
14.10	1) 1558,3; 2) 4,47; 0,22; 7 792; 3) 588,62; 8,4%; 4) a) 404,26; b) 184,36
14.21	1) 643; 661; 2) 9,3; 9,4; 0,1075; 0,1064; 9,7; 9,6; 3) -7; 25
14.23	1) 11,5; 2) 7,8

V.V. Narbut, E.P. Shpakovskaya, V.N. Salin

ECONOMIC STATISTICS

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SIA BIOM.LV
Rīga, LV-1009
Augusta Deglava iela 7 – 32

